

T H E
SEA OFFICERS COMPANION,

BEING AN

A P P E N D I X

T O

WADDINGTON'S NAVIGATION;

(WHICH MAKES IT

A COMPLETE SYSTEM OF THAT ART)

CONTAINING

Several METHODS of obtaining the LATITUDE OF A PLACE either at SEA or on
LAND, both by SUN and MOON.

THE VARIATION OF THE NEEDLE, OBTAINED BY THE MOON.

A TABLE of the LIMITS of the TIMES for taking the ALTITUDES of SUN or MOON, in
order to obtain the LATITUDE from Two ALTITUDES and the TIME ELAPSED.

The METHOD of obtaining the LONGITUDE at SEA, and of computing the
OBSERVATIONS in a concise and easy Manner.

THE NATURE AND PRINCIPLE OF REFRACTION AND REFLECTION, AS DEFINED BY
SIR ISAAC NEWTON.

TABLES OF REFRACTION, AND PARALLAX OF THE MOON AT ALL ALTITUDES,
ILLUSTRATED.

THE SOLAR SYSTEM DESCRIBED.

THE METHOD OF USING ARTIFICIAL HORIZONS BOTH AT SEA AND LAND.

The METHOD of SURVEYING COASTS and HARBOURS, with COPPER-PLATE
PLANS for the ILLUSTRATION.

By R. WADDINGTON, TEACHER of MATHEMATICS,
SURRY-SIDE, WESTMINSTER-BRIDGE.

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TO THE HONOURABLE

DIRECTORS OF THE UNITED COMPANY OF MERCHANTS
TRADING TO THE EAST INDIES.

GENTLEMEN,

IN Consequence of your Reference to the COMMITTEE of SHIPPING, I was honoured with the Permission to dedicate to You my former Edition of the Longitude, deduced from Observations of the Sun, Moon, and Stars—which Impression being fold, I have at the Desire of my Friends, in your HONOURS Employ, printed a New Edition of that Work, with many Additions and Improvements essentially necessary to be known by Your Officers, for their determining the Latitude and Longitude in stormy Weather, with Certainty, both by Sun and Moon; having for that Purpose calculated a Table of the Limits of the Times for making the Observations, and for all Declinations of Sun or Moon, and all Latitudes to 60 Degrees.—To this (together with the easy and accurate Method of obtaining the Longitude at Sea, from Celestial Observations) I have also added the Method of Surveying Coasts and Harbours with a Log-line and Azimuth Compaſs: That these, with the other Improvements, may merit Your HONOURS Approbation, and produce me Encouragement to make further Improvement in the Art of Navigation, especially for the Benefit of all Persons Trading to the East Indies, is the hearty Desire of,

GENTLEMEN,

Your most obliged,

most respectful,

and obedient Servant,

Surry-Side, Westminster-Bridge,
Sept. 30th, 1778.

ROBERT WADDINGTON.

P R E F A C E.

IN the following Tract, I have shewn every Method of obtaining the Latitude, from Observation, whereby a sufficient Degree of Accuracy may be produced—And such Methods as probably might not be productive of such Effect, I have entirely omitted; as the Mariner, by such, might be led into an Error of fatal Consequence. The Rules, and the Two Large TABLES of LIMITS in p. 4, 5, 6, being duly regarded, the Latitude may always be depended upon. The Longitude of a Ship, also will be determined with sufficient Precision by those who are properly acquainted with the Method of Calculation and Observation, with the Assistance of a Nautical Almanac and a good Octant or Sextant—the Sextant has no Advantage over the Octant except in making Observations, when the Distances of the Sun and Moon exceed 90 degrees. But as a much greater Number of Ships are conducted from one Port to another without the Help of these Methods—to those who have only their Dead Reckonings to trust to (in respect of Longitude) I recommend Care in chusing Books whose Traverse Tables are accurate, as also the Tables of Sun's Declination. By comparing some good Old Book with the New Ones, you will soon see the Cause why such bad Reckonings have been kept—as appears by many Journals of late Voyages, which I have perused.

R. W.

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A P P E N D I X.

To find the LATITUDE of a SHIP, or PLACE, by observing the
Meridian Altitude of the Moon.

Art. I. **F**IND the time that the moon passes your meridian; reduce it to the meridian of Greenwich, then find the declination of the moon at that time: or find the time, by the Nautical Ephemeris, that the moon passes the meridian of Greenwich; and then for every 15° of longitude you are West, to add 2 minutes of time, to the time of its passing that meridian; but for each 15° East, subtract 2 minutes, and the result will be the time to which the declination and the parallax of the moon are to be found.

Degrees of longitude, 15° 30 45° 60 75 90 105 120 135 150 165 180° W.

Moon passes mer. later 2' 4 6 8 10 12 14 16 18 20 22 24' Add.

But if your longitude be East, the moon passes the meridian sooner, by the minutes of time under the degrees of longitude, as above, which subtract.

Art. II. To reduce the observed altitudes of the moon to true altitudes.

Part I. Find the moon's horizontal parallax and semidiameter, by the Nautical Ephemeris for the given year.

2. Enter the table of parallaxes, with the apparent or observed altitude of the upper or lower edge, and take out the parallax of altitude. (p. 96.)

3. To the observed altitude of the lower edge of the moon, add its half diameter; but if the upper edge was observed, subtract the half diameter, and add the difference between the parallax of altitude and the refraction, and the result will be the true altitude of the moon.

When the Time of the RISING and SETTING of the MOON is required.

1. Find the declination of the moon at 6 h. before, and at 6 h. after the time of her passage over the meridian.

2. Enter the table in p. 97, with the latitude and declination, and at the angle of meeting is the time to be applied to 6 h. which will nearly give the times of the rising and setting of the moon's upper edge, before and after her passing the meridian.

Ex. 1. At sea, the 18th of Feb. 1777, the meridian altitude of the upper edge of the moon was observed to be $61^\circ. 50'$.

By Almanac, the horizontal parallax of the moon is $56'\frac{1}{2}$, and the half diameter of the moon $15'\frac{1}{2}$.

Now by Table, p. 96, with 62° altitude, and $56'\frac{1}{2}$ horizontal parallax, is $26'$, the parallax of the moon at that altitude. Also for half diameter $15'\frac{1}{2}$, and refract. $\frac{1}{2}$ is 16 subtract

The diff. is 10

To $61^\circ. 50'$

Add $00. 10$ the diff. between the parallax of alt. the refraction, and half diameter.

Gives $62. 00$ the true altitude of the center of the moon.

Z. D. $28. 00$ S.

Decl. $21. 19$ N. by Nautical Almanac.

Lat. $49. 19$ N.

Ex. 2. Suppose the merid. alt. of the moon's lower edge, the 20th of Oct. be observed $47^\circ. 56'$ S. in longitude 56° W.

Half diameter add $00. 16$

Apparent alt. of moon's center $48. 12$ S.

By Almanac, the moon on the meridian of Greenwich at 16 h. 16 m.

For 56° W. longitude, add, as per table, above 7

Moon on the meridian of the ship, at $16. 23$ P. M.

A

To

To find the declination; take the diff. between the declination of the moon at midnight of the 20th, and the noon following, which is $33'$; then,

As 12 h. is to $33'$ so is 4 h. $\frac{1}{3}$, the time after midnight, to $12'$	And $33'$
Add decl. $22^\circ. 12'$	$\times 4\frac{1}{3}$
True decl. $22. 24$ N.	$12)143(12$ nearly.

Moon's horizontal parallax $59'\frac{1}{2}$, and with 48° altitude nearly, I find by Table, p. 96, the moon's parallax at that altitude to be $39'\frac{1}{2}$, refraction $1'$, the difference $38'\frac{1}{2}$, which added to the apparent altitude of the center, gives

The true altitude $48^\circ. 50'\frac{1}{2}$ S.

Z. D. $41. 09\frac{1}{2}$ S.

Moon's declination $22. 24$ N. at 16 h. 23'. after the noon of the 20th day.

Lat. $63. 33\frac{1}{2}$ N.

Ex. 3. The 11th of December, 1777, at what time will the moon rise and set, in the latitude $47^\circ. 40'$ N. and longitude 106° E. the moon passes the meridian of Greenwich at 9 h. 30 m. P. M. moon's declination 12° N. nearly? By Table, p. 97, under 48° , and against 12° decl. is 0 h. 55 m. to be added to 6 h. which gives 6 h. 55 m. the semi-diurnal arch of the moon above the horizon. Hence it is manifest that time must be subtracted from the time of the moon passing the meridian, the remainder will be the time of her rising nearly; and that quantity added to her time of passing the meridian, will be the time of setting.

Thus 9 h. 30 m.

Subtract and add	6 . 55	
Time of moon's rising	2 . 35	} P. M.
Time of setting	16 . 25	

To find the DECLINATION of the MOON, when she passes that Meridian which is 106° E. from Greenwich.

The time of passing the meridian is	9h. 30'	Moon's declination at noon	$9^\circ. 27'$ N.
For 106° E. subtract	0 . 14	At midnight	$12. 05$ N.
Moon passes merid. of the place at	9 . 16	Increase in 12 h.	2 . 38
The moon's declination then	11 . 29 N.	in 6	1 . 19
* Do. at rising	9 . 57	in 3	0 . 40
Do. at setting	13 . 01	for 16'	0 . 03
		Increase in 9 h. 16 =	2 . 02
		Add to declination at noon	9 . 27
		True declination	11 . 29 N.
For 7 h. before and after passing the meridian, subtract and add	1 . 32		
		Diff.	9 . 57
		Sum	13 . 01 *

To find the Time of the RISING and SETTING of the MOON, at the Place 106° E.

By Almanac, moon passes the meridian at	9h. 16'
By Table, under the latitude, and against the declination, 10° is	6 . 45 sub.
And against the declination at the time of setting 13 is	6 . 59 add
Therefore the time of rising is	2 . 31 P. M.
And of setting	16 . 15

Art. III. To find the VARIATION of the NEEDLE by the Moon.

Make the observations in all respects as with the sun; and reduce the mean of the observed altitudes, to the true altitude.—Also find the true declination of the moon to the time

time (as in Art. I.) then compute the true azimuth, as in the sun's, by Rule 2. p. 100. And note, when the moon is in the Eastern part of the horizon, to find which way the needle varies, by the general Rule in p. 90, Art. 5.

N. B. The variations may be obtained by the moon, nearly as well as by the sun, which I have experienced by a considerable number of observations, both at sea and on shore.

At Portsmouth, in September, 1764, I found the variation, by the mean of eight observed azimuths of the sun, to be $19^{\circ} 35' \text{ W.}$

And by the mean of 7 by the moon $19^{\circ} 40' \text{ W.}$

Oct. 1770, the variation at London $20^{\circ} 25'$

at the Nore $20^{\circ} 05'$

at the Spurn $22^{\circ} 00'$

A New METHOD to find the LATITUDE at SEA, with as much Certainty as by a Meridian observation, provided Three Altitudes of the Sun can be taken at equal Intervals of Time, under the Circumstances noted in the following Table.

Mer. Alt.	Intervals of Time,		The nearest Obser. to or from * Noon not to exceed	
	not less than	not greater than		
0			h	
65	6	10	0	15
60	8	12	0	18
55	10	15	0	24
50	12	20	0	30
45	14	24	0	36
40	16	28	0	42
35	18	30	0	50
30	20	34	1	0
25	20	36	1	0
20	20	40	1	0

Since the meridian altitude may always be known within the limits of the first column; you will therefore know the proper intervals of time to be taken between the observations, as noted in the 2d and 3d columns. You may likewise generally know, whether your nearest observation to the meridian happens within the limits of the 4th col. which particulars corresponding, then

Note, If the ship does not alter her course or rate of sailing considerably, during the time of the observation, no allowance for her motion need be made.

GENERAL RULE.

Take the least altitude from each of the other two, then from 4 times the lesser difference, take the greater, and from the remainder take the greater difference, and then divide the square of the first remainder, by 4 times the 2d remainder,

the quotient will be the minutes to be added to the least altitude, which gives the meridian altitude.

Remark I. When the 1st and 3d altitudes happen to be equal, the 2d altitude is the meridian altitude.

Remark II. When the two first, or the two last (of the three observations) happen to be equal, the instant of noon is the middle time between them; then add $\frac{1}{2}$ part of the diff. between one of those equal altitudes, and the least, to one of the equal altitudes, the sum will be meridian altitude.

Ex. I. 1770, per Watch, Sept. 16th.

By watch, at 11h. 0' Alt. Sun $41^{\circ} 27'$

11 . 20 $42^{\circ} 23'$

11 . 40 $42^{\circ} 59'$

Lesser diff. $0^{\circ} 56'$ times 4 is 224

Greater diff. $0^{\circ} 92'$ 92

First remainder 132 squared is 17424

-92

Second remainder 40 times 4 is 160

By

* Time to Noon, is the time before Noon; and time from Noon, is the time after noon.

By General Rule.

160)17424(109' = 1°. 49' to which	
Add the least alt.	41 . 27
Mer. alt. of the Sun	43 . 16
Z. D.	46 . 44 S.
Decl.	2 . 33 N.
Lat.	49 . 17 N.

Ex. 2. At 11h. 50'—43h. 12'

12 . 10 — 43 . 12

12 . 30 — 42 . 40

 $\frac{1}{8}$ 32

By Art. 2d, Add 0 . 4

Sun's mer. alt. as before 43 . 16

A TABLE of the Hours of Time (before Noon and after Noon) that the Altitudes of the Sun may be taken, in order to obtain the Latitude, by means of two Altitudes, and the Time elapsed between the Observations, when the Lat. and Decl. are of the same Name.

Lat.	Dec. 1°	Dec. 3°	Dec. 5°	Dec. 7°	Dec. 9°	Dec. 11°	Dec. 13°	Dec. 15°	Dec. 17°	Dec. 19°	Dec. 21°	Dec. 23°
°	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.
2	0 . 6	0 . 6	0 . 20	0 . 35	0 . 54	1 . 7	1 . 21	1 . 40	1 . 58	2 . 20	2 . 38	3 . 0
4	0 . 20	0 . 6	0 . 6	0 . 24	0 . 42	0 . 58	1 . 14	1 . 30	1 . 48	2 . 8	2 . 28	2 . 48
6	0 . 25	0 . 20	0 . 5	0 . 10	0 . 28	0 . 40	0 . 55	1 . 15	1 . 35	1 . 50	2 . 10	2 . 35
8	0 . 50	0 . 35	0 . 20	0 . 7	0 . 0	0 . 24	0 . 40	0 . 55	1 . 1	1 . 30	1 . 50	2 . 10
10	1 . 0	0 . 45	0 . 33	0 . 20	0 . 7	0 . 7	0 . 20	0 . 35	4 . 52	1 . 10	1 . 30	1 . 50
12	1 . 15	1 . 0	0 . 48	0 . 35	0 . 20	0 . 6	0 . 6	0 . 25	0 . 40	1 . 0	1 . 20	1 . 40
14	1 . 25	1 . 10	1 . 0	0 . 46	0 . 33	0 . 22	0 . 6	0 . 6	0 . 66	0 . 45	1 . 3	1 . 23
16	1 . 40	1 . 27	1 . 10	1 . 0	0 . 48	0 . 34	0 . 22	0 . 6	0 . 6	0 . 25	0 . 40	1 . 0
18	1 . 42	1 . 30	1 . 20	1 . 8	0 . 53	0 . 42	0 . 30	0 . 20	0 . 7	0 . 7	0 . 23	0 . 45
20	1 . 55	1 . 44	1 . 32	1 . 22	1 . 10	1 . 0	0 . 48	0 . 33	0 . 20	0 . 7	0 . 7	0 . 26
22	2 . 5	1 . 54	1 . 43	1 . 32	1 . 20	1 . 8	0 . 56	0 . 44	0 . 32	0 . 20	0 . 7	0 . 7
24	2 . 10	2 . 0	1 . 50	1 . 40	1 . 28	1 . 18	1 . 6	0 . 54	0 . 42	0 . 30	0 . 20	0 . 7
26	2 . 20	2 . 10	2 . 0	1 . 51	1 . 41	1 . 30	1 . 20	1 . 8	0 . 56	0 . 43	0 . 32	0 . 20
28	2 . 28	2 . 18	2 . 8	1 . 58	1 . 48	1 . 40	1 . 30	1 . 20	1 . 8	0 . 56	0 . 45	0 . 33
30	2 . 34	2 . 28	2 . 15	2 . 6	1 . 58	1 . 50	1 . 40	1 . 30	1 . 20	1 . 10	0 . 55	0 . 45
32	2 . 42	2 . 34	2 . 25	2 . 15	2 . 7	1 . 57	1 . 48	1 . 40	1 . 30	1 . 20	1 . 10	1 . 0
34	2 . 50	2 . 42	2 . 32	2 . 23	2 . 14	2 . 7	2 . 0	1 . 50	1 . 40	1 . 30	1 . 22	1 . 10
36	2 . 55	2 . 46	2 . 38	2 . 30	2 . 22	2 . 15	2 . 6	1 . 56	1 . 48	1 . 40	1 . 30	1 . 20
38	3 . 0	2 . 50	2 . 42	2 . 36	2 . 28	2 . 20	2 . 12	2 . 2	1 . 56	1 . 48	1 . 38	1 . 28
40	3 . 6	2 . 56	2 . 48	2 . 42	2 . 34	2 . 27	2 . 20	2 . 12	2 . 5	1 . 56	1 . 48	1 . 40
42	3 . 10	3 . 2	2 . 55	2 . 48	2 . 40	2 . 32	2 . 26	2 . 20	2 . 12	2 . 4	1 . 56	1 . 50
44	3 . 15	3 . 8	3 . 0	2 . 56	2 . 46	2 . 40	2 . 33	2 . 27	2 . 20	2 . 12	2 . 4	2 . 0
46	3 . 18	3 . 13	3 . 7	3 . 0	2 . 52	2 . 45	2 . 40	2 . 34	2 . 28	2 . 20	2 . 12	2 . 6
48	3 . 22	3 . 18	3 . 11	3 . 5	2 . 58	2 . 52	2 . 45	2 . 40	2 . 34	2 . 28	2 . 20	2 . 13
50	3 . 27	3 . 22	3 . 16	3 . 10	3 . 4	2 . 58	2 . 53	2 . 46	2 . 40	2 . 35	2 . 28	2 . 20
52	3 . 30	3 . 25	3 . 20	3 . 13	3 . 8	3 . 2	2 . 57	2 . 52	2 . 46	2 . 40	2 . 35	2 . 28
54	3 . 32	3 . 27	3 . 22	3 . 16	3 . 12	3 . 6	3 . 0	2 . 56	2 . 50	2 . 45	2 . 40	2 . 35
56	3 . 35	3 . 32	3 . 25	3 . 20	3 . 16	3 . 10	3 . 5	3 . 0	2 . 55	2 . 50	2 . 45	2 . 40
58	3 . 38	3 . 34	3 . 30	3 . 25	3 . 20	3 . 14	3 . 9	3 . 5	3 . 0	2 . 55	2 . 50	2 . 46
60	3 . 40	3 . 36	3 . 32	3 . 27	3 . 22	3 . 18	3 . 12	3 . 8	3 . 4	3 . 0	2 . 56	2 . 52

Note. In the table, at the angle of meeting, under the declination, and against the latitude, is the time, before or after Noon, when the position, or azimuth of the sun, does not exceed 60° from the meridian, or $5\frac{1}{2}$ points; within which limits the altitudes should be taken; and the nearer one of the latitudes is taken to or from noon the better.

A TABLE of the Limits of the Time continued, the Latitude and Declination of contrary Names.

Lat.	Dec. 1°	Dec. 3°	Dec. 5°	Dec. 7°	Dec. 9°	Dec. 11°	Dec. 13°	Dec. 15°	Dec. 17°	Dec. 19°	Dec. 21°	Dec. 23°
°	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.
0	0. 6	0. 20	0. 36	0. 50	1. 8	1. 22	1. 40	1. 55	2. 12	2. 32	2. 54	3. 18
2	0. 20	0. 34	0. 48	1. 2	1. 18	1. 30	1. 47	2. 2	2. 20	2. 40	3. 2	3. 24
4	0. 33	0. 45	0. 58	1. 13	1. 29	1. 40	1. 55	2. 12	2. 28	2. 50	3. 10	3. 32
6	0. 45	1. 0	1. 10	1. 25	1. 40	1. 52	2. 10	2. 25	2. 40	3. 0	3. 20	3. 40
8	1. 0	1. 12	1. 27	1. 40	1. 55	2. 8	2. 25	2. 40	2. 55	3. 10	3. 30	3. 46
10	1. 10	1. 27	1. 40	1. 50	2. 7	2. 20	2. 36	2. 50	3. 8	3. 20	3. 38	3. 54
12	1. 25	1. 40	1. 50	2. 5	2. 20	2. 30	2. 45	3. 0	3. 15	3. 30	3. 48	4. 4
14	1. 40	1. 50	2. 0	2. 12	2. 28	2. 40	2. 54	3. 10	3. 25	3. 40	3. 56	4. 13
16	1. 48	2. 0	2. 10	2. 27	2. 38	2. 50	3. 3	3. 16	3. 32	3. 48	4. 4	4. 20
18	1. 55	2. 10	2. 20	2. 30	2. 45	2. 57	3. 10	3. 23	3. 38	3. 54	4. 10	4. 25
20	2. 5	2. 18	2. 28	2. 40	2. 52	3. 5	3. 18	3. 30	3. 44	4. 0	4. 15	4. 30
22	2. 15	2. 25	2. 36	2. 47	3. 0	3. 10	3. 22	3. 35	3. 48	4. 2	4. 18	4. 33
24	2. 20	2. 30	2. 42	2. 54	3. 5	3. 15	3. 27	3. 40	3. 52	4. 5	4. 22	4. 36
26	2. 30	2. 40	2. 50	3. 0	3. 13	3. 25	3. 35	3. 44	3. 56	4. 8	4. 25	4. 38
28	2. 36	2. 48	2. 56	3. 7	3. 18	3. 29	3. 38	3. 48	4. 0	4. 12	4. 27	4. 40
30	2. 44	2. 54	3. 3	3. 14	3. 24	3. 33	3. 43	3. 52	4. 3	4. 15	4. 28	4. 40
32	2. 50	3. 0	3. 10	3. 20	3. 28	3. 36	3. 46	3. 56	4. 6	4. 18	4. 30	4. 40
34	2. 55	3. 5	3. 15	3. 25	3. 32	3. 40	3. 50	4. 0	4. 9	4. 20	4. 30	4. 40
36	3. 0	3. 10	3. 20	3. 28	3. 36	3. 44	3. 52	4. 2	4. 10	4. 21	4. 30	4. 40
38	3. 6	3. 15	3. 23	3. 32	3. 38	3. 46	3. 55	4. 4	4. 12	4. 22	4. 32	4. 40
40	3. 12	3. 20	3. 28	3. 35	3. 43	3. 50	3. 58	4. 6	4. 14	4. 23	4. 32	4. 42
42	3. 16	3. 23	3. 30	3. 38	3. 46	3. 52	4. 0	4. 8	4. 15	4. 24	4. 32	Set.
44	3. 22	3. 28	3. 35	3. 42	3. 49	3. 5	4. 2	4. 10	4. 17	4. 25	4. 33	
46	3. 26	3. 33	3. 40	3. 46	3. 52	3. 58	4. 5	4. 11	4. 18	4. 26	4. 33	
48	3. 30	3. 36	3. 42	3. 48	3. 54	4. 0	4. 6	4. 12	4. 18	4. 26	Set.	
50	3. 33	3. 38	3. 44	3. 50	3. 55	4. 0	4. 6	4. 12	4. 19	4. 26		
52	3. 36	3. 40	3. 46	3. 52	3. 57	4. 1	4. 7	4. 13	4. 19	4. 26		
54	3. 38	3. 42	3. 48	3. 53	3. 58	4. 2	4. 8	4. 13	4. 20	Set.		
56	3. 40	3. 44	3. 50	3. 54	4. 0	4. 3	4. 9	4. 13	4. 20			
58	3. 43	3. 46	3. 51	3. 55	4. 0	4. 4	4. 10	4. 14	Set.			
60	3. 45	3. 48	3. 52	3. 56	4. 1	4. 5	4. 11	4. 14				

Note, When one of the altitudes is made near the utmost limits of the time, as in the column, that the interval of time should be an hour or more; and the nearer one of them is taken to noon the better.

To find the lat. (of a ship at sea, or of a place) by observing two altitudes of the sun, and the time elapsed between the observations.—Assume the lat. from the ship's reckoning, as near as you can, and find the declination of the sun to the time of the greater altitude; then find the half sum of the co-lat. polar dist. and the great zenith dist. and subtract the co-lat. from the half sum. Subtract also the polar dist. from it; and then observe these

GENERAL CANONS.

Canon 1. To the fines co-ar. of the co-lat. and polar dist. add the fines of the two differences, contained between the half sum and the co-lat. and polar dist. and half the sum of these logarithm fines, will be the fine of half the angle of time, to or from noon, when the less alt. was taken.

B

Canon.

Note, Dec. for Declination:

Canon 2. Say, as $R : \text{co-t. of the decl.} :: \text{co-f. of the angle of the time to or from noon of the greater alt.} : t. \text{ of the dist. of the perpendicular from the pole.}$

Note 1st. If the latitude and declination be of different names, then the polar dist. and the distance of the perpendicular from the pole is greater than 90° . In which case the supplement of its distance, by the 2d Canon, is to be taken, as in the above example.

Note 2d. By polar dist. is meant the degrees and minutes the sun is from the pole of the same name as the latitude; and when the polar dist. exceeds 90° , the perpendicular also exceeds 90° ; therefore the supplement will be its distance.

Note 3d. When both altitudes are taken before or after noon, then from the angle of the time of the less altitude to be the time to or from noon, when the greater altitude was taken. But if one observation be made in the forenoon and the other in the afternoon, then the time of the less altitude, taken from the elapsed time, gives the time of the greater altitude.

Canon 3. Say, as $S. \text{ decl.} : \text{to co-f. polar dist. of the perpendicular} :: \text{co-f. of the less zenith dist.} : \text{the co-f. of the perpendicular from the zenith.}$

Then from the polar dist. of the perpendicular, take the dist. of the perpendicular from the zenith, and the co-lat. will remain.

Ex. In the Lat. $51^\circ. 30' \text{ N.}$ The sun's decl. $18^\circ. 30' \text{ S.}$
By ship acc. 51.26 Sun's polar dist. 108.30

per Watch At 1h. 0' P.M. Zenith distance $71^\circ. 14'$
At 1.40 Zenith distance 73.20

Elapsed time $0.40 = 10^\circ$ the angle of the elapsed time.

Co-lat.	$38^\circ. 34'$	$0,205216$
Polar dist.	108.30	$0,023043$
The greater Z.D.	73.20	$9,977293$
Sum	220.24	$8,472263$
$\frac{1}{2}$ Sum	110.12	$18,677815$
Diffs.	$\left\{ \begin{array}{l} 71.38 \\ 1.42 \end{array} \right.$	$\left\{ \begin{array}{l} 9,338907 \\ \text{s. of } 12^\circ.36\frac{1}{2} \end{array} \right.$

Angle of time from noon at least alt. 25.13 | 2d, $R : \text{co-tan. of } 18^\circ. 30' - 10,475480$
Deduct the angle of time elapsed 10.0 | $:: \text{co-f. of } 15.13 - 9,984500$

Angle of time from noon at gr. alt. 15.13 | $: t. \text{ of } 70.52\frac{1}{2} = 10,459980$

Which sup. is the dist. of perp. from the pole $109.7\frac{1}{2}$

3d. s. of decl.	$18^\circ. 30'$	$0,498524$
: co-f. of	$70.52\frac{1}{2}$	$9,515384$
:: co-f.	71.14	$9,507471$
: co-f. of	70.36	$9,521379$
Polar dist of perp.	$109.07\frac{1}{2}$	
The diff.	$38.31\frac{1}{2}$	is the co-lat.
Hence the lat.	$51.28\frac{1}{2}$	N.

Note, If the lat. resulting be greater than that assumed, add the double of the difference to that assumed: but if that resulting be less, subtract the double difference, and you will have the lat. nearly. Then make your 2d assumption and computation. By assuming $10'$ too much, the result was $9'$ less than that assumed; and by assuming $10'$ too little, the result was $6'$ more than that assumed.

Assumed lat. $51^\circ. 20'$ the result is $51^\circ. 26'$

Add half the difference $2\frac{1}{2}$

Lat. $51.28\frac{1}{2}$

The true lat. was 51.30 N.

REMARKS!

Note, Z.D. for Zenith distance, or co-alt,

(7)
R E M A R K S.

I. When the latitude is assumed too little, and one alt. taken before, and the other after noon, the result will be too much; when it is assumed too much, the latitude resulting will be too little; but the contrary, when both altitudes are taken before, or both after noon.

II. When the ship differs her latitude (between the observations) towards the sun, add that difference of latitude to the first altitude, and the place of observation will be reduced to that where the 2d altitude was taken: But if subtracted, it will be reduced to the place of the first altitude, and the contrary.

III. When the ship differs her longitude East, add the minutes of longitude made between the observations, to the elapsed angle of time; but if you have made West longitude subtract it, and you will have the angle of elapsed time corrected, with which you are to work, as follows:

Suppose the time of 1st alt. 11h. 0' A. M.
of 2d 1. 40 P. M.
Elapsed time 2. 40 = 40°. 00' the angle of elapsed time
 $\angle$ of time from noon at the least alt. 25. 13 by the foregoing Ex.
 $\angle$ of time from noon at the gr. alt. 14. 47
This $\angle$ of time by 2d Canon gives 70. 55
Its sup. 109. 05
By 3d Canon 70. 38
Co-lat. 38. 27 which is 3' too little
Therefore the lat. is 51. 33 or 3' too much

Since repeating the work, with a new assumption of the latitude, is troublesome, and in some circumstances very defective, which makes the solar tables too often err half a degree, or more; and the same by the foregoing method; I would therefore recommend the following method, by which the latitude will be nearly obtained at the first operation, and by using only the 2d and 3d of the foregoing Canons.

First, I suppose your watch to go solar time, or nearly so; being thus prepared you are to take the altitude of the sun, when near the prime vertical, or when it rises or falls very quick, and find the time by that altitude; whereby you will find how much your watch is fast or slow, for the sun.

Ex. Suppose the alt. of the sun taken at 8h. 12' A. M. by the watch
And the time by the sun's alt. 8. 2
0. 10

Note, That by assuming the latitude 20 or 30' more or less than the true lat. the time resulting, will be very near the same, when the altitudes are taken near the prime vertical, therefore the times may be well determined, although the latitude is not known nearer than 20'.

The altitudes of this Ex. were (by the watch) taken at 11h. 10'. A. M. and 1h. 50' P. M.

Which reduced to true time is 1h. 00' before noon = 15°

And the other at 1. 40 after noon = 25

The sum is the $\angle$ of time 40 or the $\angle$ of time elapsed,

R : co-tan. decl.	18°. 30' = 10,47541
:: Co-f. $\angle$ of time	0. 15 — 9,98494
: t. of	70. 54
Sup.	109. 6 — 10,46842
s, Decl. co-ar.	18. 30 — 0,49852
Co-f.	70. 54 — 9,51484
Co-f.	71. 14 — 9,50747
Co-f.	70. 37½
Sup. from above	109. 6 — 9,52083
Diff. is co-lat.	38. 28½ Lat. 51°. 31½N.

Note, When the result of the 2d Canon, is the same as the polar dist. then the zenith distance is the meridian zenith distance.

The

The 13th of March, 1775, in the lat. of $51^{\circ} 29' N$. I observed the following altitudes of the moon.

Time by the watch at 7h.	8'	moon's alt.	$40^{\circ} 30'$	— 1'	ref. H par.	$54\frac{1}{2}'$	par. of alt.	$41'$
7 . 24	—	—	$42 . 25$	— 1	ref. H par.	do.	do.	$40'$
. 8 . 28	—	—	$49 . 02$	— 1	+ 35'	parallax of altitude.		
8 . 32	—	—	$49 . 23$	— 1	+ $34\frac{1}{2}'$			
. 9 . 30	—	—	$52 . 30$	— 1	+ 33			
9 . 35	—	—	$52 . 39$	— 1	+ $32\frac{1}{2}'$			
		Mer. alt.	$52 . 48$	— 1	+ $32\frac{1}{2}'$			
		For parx. and ref. +	$31\frac{1}{2}'$					
			$53 . 19\frac{1}{2}'$					
		Z. D.	$36 . 40\frac{1}{2}' S$.					
		Decl.	$14 . 47\frac{1}{2}' N$.					
		Lat.	$51 . 28' N$.					

March 14th, Moon's mer. alt. $49^{\circ} 52'$

For parallax and refraction + $32'$

True alt. $50 . 24$

Zen. dist. $39 . 36 S$.

Decl. $11 . 54 N$.

Lat. $51 . 30 N$.

Ditto 13th $51 . 28 N$.

The mean is the true lat. $52 . 29 N$.

N. B. The hourly difference of declination is $6'$ decreasing.

To find the Difference of the Altitudes of the Moon, occasioned by the Alteration of its Declination, Parallax and Refraction.

Let the altitudes observed (that are to be used) be those taken at 8h. 28', which is $49^{\circ} 36'$; and at 9h. 30', is $52^{\circ} 2'$; the alteration of declination between 8h. 28' and 9h. 30' is $6'$. Hence,

At 8h. 28' Decl. $14^{\circ} 54' N$.

At 9 . 30 Decl. $14 . 48 N$.

At the Middle time is 8 . 59 and $14 . 51 N$. Decl.

Polar dist. $75 . 09$

Elapsed time 1 . 02 = $15 . 30 \angle$ of Time.

The parallax of altitude, corresponding with $3'$ alteration of declination, may be taken $2'$; for it will always be less than the declination, except when it is reckoned upon the meridian; now these $2'$ taken from the first alt. and added to the 2d gives the altitudes, $49^{\circ} 34'$, and $53^{\circ} 04'$, and the zenith dist. $36^{\circ} 56'$, and $40^{\circ} 26'$.

N. B. The reason why $2'$ are taken, &c. See 2d Edit. of the Sea-Officer's Companion.

Co-lat. $38^{\circ} 31'$	— co-ar. 0,205692	R : co-t. decl. $14^{\circ} 51'$	— 10,570516
Polar dist. $75 . 09$	— co-ar. 0,014753	:: co-f. $\angle$ time $5 . 51$	— 9,997732
Gr. Z. D. $40 . 26$	— 9,794467	: t. of $75 . 04\frac{1}{2}$	— 10,574248
$154 . 06$	— 8,520551	f. decl. $14 . 51$	— 0,591270
$77 . 03$	18,535463	: co-f. of $75 . 04\frac{1}{2}$	— 9,410869
$38 . 32$	9,267732	:: co-f. of $36 . 56$	— 9,902729
$1 . 54$	f. of $10^{\circ} 40'\frac{1}{2}$	: co-f. of $36 . 33$	— 9,904868
Its double is the $\angle$ of time of		Co-lat. $38 . 31\frac{1}{2}'$	
the less alt.	$21 . 21$	Lat. $51 . 28\frac{1}{2}' N$.	
Subtract the $\angle$ of elapsed time	$15 . 30$		
Rem. $\angle$ of time (of moon to			
mer.) at the great alt.	$5 . 51$		

I have here only given one example, to shew that the latitude may be easily obtained by two altitudes of the moon and the time between them, as well as with the sun; of which see the method clearly shewn in my Sea-Officer's Companion, 2d Edit. Of

Of the METHOD for determining the LONGITUDE at SEA.

BEING provided with a good octant or sextant, which has a telescope and moon glasses, adapted to darken the moon's light, so as to make it nearly equal with that of the star, by which the distance is to be taken at the limb of the moon, or in contact with her full limb, by reflection.

1. The many different circumstances which happen respecting these observations, and the different degrees of light of the moon and stars; by which many observations prove abortive. I have, by repeated trials, at last, got over this obstacle.

2. There is another impediment; which is, the light of the moon that falls on the eye, when you are observing the distance of her from stars that do not exceed 50° . This imperfection I have also taken off by a shade to the eye.

3. It frequently happens, that the distance of the moon and star can be observed when neither the altitude of the moon or star can be taken; in which case, the time being well ascertained, their altitude may be computed.

4. To find the apparent time of an observation, is a material circumstance; and it frequently happens that you must trust to the going of your watch for some hours before or after the time that you observe the distance; especially in the night, when you observe the dist. of the moon and star.

5. The proper times of the day for taking the altitudes of the sun, in order to find the true time (very near), is in the fore or afternoon, when the bearing of the sun is between five and eight points from the meridian, and the nearer to the prime vertical, or East and West, the better; because in these positions he rises and falls the quickest.

To find the **TRUE TIME**, by observing the **ALTITUDE** of the **SUN**.

6. The method is to take two or three altitudes of the sun, and to note the time by the watch at each altitude. These altitudes should be taken at two, three or four minutes after each other; then take the mean. **N. B.** To take the mean, is to add the hours, minutes and seconds together, and to divide the sum by the number of observations, the result will be the mean or middle time: From the mean altitude take the refraction and dip, the remainder will be the true altitude of the sun; and its complement the true **Z.D.** or co-alt. Do the like with all altitudes and distances; then compute the time by the following General Canon.

From half the sum of the co-lat. polar distance and co-alt. take the co-lat. and polar distance separately, and note the remainders; then to the sines co-ar. of the co-lat. and polar distance, add the sines of the two remainders; the half of the sum of these sines will be the sine of half the angle of the time before or after noon.

7. At the time the distance of the sun and moon is observed, their altitudes are generally taken by two assistants, and there should be three or four altitudes of each taken, and as many distances at the same times; and these observations should be made at four, five or six minutes after each other; then reduce the several times, the altitudes of the moon, and them of the sun or star; as also of the distances into one observation of time, one altitude of each object, and one distance, by finding the mean of each; and reduce the mean time of the observation by the watch, to the true time of the observation, by adding or subtracting what the watch was too fast or too slow.

To calculate the **ALTITUDE** of the **SUN**, **MOON** or **STAR**.

8. Having given the latitude of the place, the declination and the time, the altitude may be found as follows:

First, For the altitude of the sun: Reduce the hours, minutes and seconds of time, before or after noon, into degrees, minutes and seconds, which will be the angle of time. Thus

for each hour reckon 15° . Also the minutes of time divided by 4, give degrees; the seconds of time divided by 4, give minutes; and if 1, 2 or 3 minutes or seconds remain, they will produce 15, 30 or 45 minutes or seconds for the angle of time: For 1 hour of time corresponds with 15° of terrestrial motion of the equator.

To reduce DEGREES and MINUTES into TIME.

9. Divide the degrees by 15, the quotient will be hours; multiply the remainder by 4, and divide the minutes of a degree by 15, for minutes of time; the remainder multiply by 4, for seconds of time.

GENERAL CANONS for finding the ALTITUDE of the SUN, MOON or STAR.

10. 1st, $R : \text{cs. of the angle of time} :: \text{ct. of the latitude} : \text{the t. of the 4th arc}$.

From the polar distance take 4th arc, and the 5th arc will remain: But when the angle of time exceeds 90° (or when the time is before 6 in the forenoon, or after 6 in the afternoon) add the 4th arc to the polar distance, the sum will be the 5th arc.

2d, $\text{Cs. 4th arc} : \text{cs. 5th arc} :: s, \text{lat.} : s, \text{true alt.}$ From the true altitude of the moon, take the difference between the parallax of her altitude and the refraction, the remainder will be her apparent altitude; but to the true altitude of the sun or star, add the refraction; the sum will be the apparent altitude.

Note. When the altitude of a star is to be found, the first thing to be done is to find its right ascension at the time of the observation. The following Table shews the right ascension and declination of such stars as are used in the observations for determining the longitude at sea.

January 1, 1775.	Right Ascension in deg.			Right Ascension in Time.			Annu. Var.	Declination.			Annual Var.
	deg.	mi.	sec.	h.	m.	s.	sec.	deg.	m.	s.	sec.
1. Arietis, the 5th * in Aries	28	37	59	1	54	32	3.337	22	23	23.6	N + 17.64
2. Aldebaran, the S. eye of the Bull	65	45	28	4	23	2	3.427	16	2	24.7	N + 8.32
3. Pollux, the Southern Twin	112	53	8	7	31	32 $\frac{1}{2}$	3.751	28	33	7.37	N - 7.72
4. Regulus, or Cor Leonis	149	5	41.5	9	56	23	3.240	13	3	32.2	N - 17.17
5. Spica Virginis, the virgin's spike	198	20	33.2	13	13	22.2	3.151	9	58	48.8	S + 18.97
6. Antares, or Cor Scorpionis	243	54	44.7	16	15	39	3.661	25	54	47.4	S + 8.89
7. Atair. mid. * of Aquila	294	56	53.8	19	39	47.6	2.903	8	17	11.	N + 8.40
8. Capric. or Capricorni	301	59	16.4	20	7	57.	3.378	15	29	50.2	S - 10.60
9. Fomalhaut	341	17	41.	22	45	10.7	3.353	30	48	29.4	S - 18.97
10. Pegasi. or Markab	343	23	33.7	22	53	34.2	2.983	13	59	54.	N + 19.20

To find the ANGLE of TIME contained between the MERIDIAN the STAR is upon at the Time of OBSERVATION and the MERIDIAN of the PLACE.

From the right ascension of the star take the right ascension of the sun at the time of observation (adding 24 h. to the star's right ascension when it is less than the sun's), the remainder will be the time of the star's passage over the meridian, from which, take the time of observation, if it be the lesser; or, from the time of observation, take the time of the star's passage over the meridian; and the difference will be the time the meridian of the star is to or from the meridian of the place; which convert into degrees, &c. by Art. 8 above.

Remark 1. The time of observations made in the forenoon, are to be reckoned from the noon of the foregoing day.

2. When the place of observation is supposed or assumed to be East of the meridian of Greenwich; then, from the time of observation, subtract 4 minutes of time for each degree of longitude; but for West longitude add, and you will have the time at Greenwich; to which meridian the tables of time, right-ascension and declination, and the distances of the sun, moon and stars, are calculated.

3. When

Note. Co-ar. signifies complement arithmetical, co-lat. comp. of lat. and co-alt. comp. of altitude.

3. When the time of observation is greater than that of the passage of the star over the meridian, the star has passed the meridian; but when it is less, it has not passed.

4. To make all observations of the distances of the moon and star, at the West limb of the moon from change to full; but after full moon, make them at the East limb, or on the left side of her.

5. When a telescope is used which inverts, you must observe the star on the contrary side to appearance without the telescope.

6. From change to full moon, add half her diameter to the distance of the stars observed West of her; but subtract half her diameter from the distance of stars East of her: After full moon use the contrary. Add also as per Table, p. 15, for app. incr. of sun's half diameter.

Having given the apparent altitudes and distance, to find the difference of the azimuths of the sun and moon, or moon and star.

GENERAL RULES.

Rule 1. To work by apparent distance (of centers), and the apparent zenith distances.

From half the sum of the apparent distance, and the two zenith distances, take the zenith distances separately, and note the remainders; then take the sine complement arithmetical of each zenith distance, and the sines of the two remainders; add the four logs together; and half the sum will be the sine of half the angle of azimuth, which double.

To correct the APPARENT ZENITH DISTANCE.

From the moon's zenith distance take the difference between the parallax of the moon's altitude and the refraction; the remainder will be the true zenith distance.

Add the refraction of sun or star to its apparent zenith distance, the sum will be the true Z. distance.

Rule 2. To work by the true zenith distances.

R : t. of the lesser of the true zenith distances :: cs. of the difference of azimuth : the t. of the 4th arch. If the difference of azimuth be less than 90° , then from the greater true zenith distance take the 4th arch, and the remainder will be the 5th arch; but if the difference of azimuth exceeds 90° , then to the 4th arch add the greater true zenith distance, and the sum will be the 5th arch.

Note, When the 5th arc is less than 90° , the reduced distance will be less than 90° ; but when the 5th arc exceeds 90° , the reduced distance will be greater than 90° ; in which case subtract the degrees and minutes from 180° , and the remainder will be the reduced dist.

Rule 3. As cs. 4th arch : cs. 5th :: cs. less true zenith distance : cs. of the true or reduced distance by observation; which being done, find the distance by the almanac, at the time of the observation, reduced to the time at Greenwich, and then;

Remark 1. When the distance is increasing, and the reduced distance, greater than that by the Ephemeris, the longitude is West; but if it be less, it is East longitude: which must be applied to the longitude that you assumed the ship to be in.

Remark 2. When the distance is decreasing, and the reduced distance is the greater, then the longitude resulting from the difference, is East; but if it be less, the longitude is West of the place assumed.

To find the Distances of the SUN and MOON, or MOON and STAR, at the time of an observation, by the Ephemeris or Nautical Almanac.

The calculated distances in the first column of the Ephemeris are adapted to noon, at the meridian of Greenwich, and the other columns contain the distances, at 3, 6, 9, 12, 15, 18, and 21 hours, after the preceding noon; and all observations must be reckoned in like manner, as by remark 1, 2, in p. 2.

Note

* By reduced distance, is to be understood the true distance by the observation; or the observed distance cleared from the effects of refraction and parallax.

Note 1. If the true time of observation at the ship, reduced to the meridian of Greenwich (by Remark 2, p. 10,) be betwixt noon and 3 p.m. then take out the distance in the column under noon, against the day of the month; and under it write the distance at 3 h. and the difference of these distances, and then say, As 3 hours, or 180 minutes, is to that difference so is the minutes of time of the observation, after the time of the first distance, to the proportional part of increase or decrease of distance; which must be added to the distance at the former time, when the distance is increasing; but subtracted when the distance is decreasing, and you will have the distance at the time of observation by the Ephemeris.

Note 2. You are always to take from the Ephemeris, one distance at the nearest time preceding that of the reduced time of observation. [See remark p. 10,] and the next distance after it, and then proceed as above directed.

Note 3. Having obtained the diff. between the reduced dist. and that by the Eph. the say, As the hourly incr. or decr. of dist. obtained from the Eph. is to 15° or $900'$ of lon. so is the difference between the reduced dist. and that by the Eph. to the degrees, or minutes of lon. resulting.

The use of the Table of the Moon's Parallax of alt. is to obtain the true altitude (inference to the centre of the earth) from the apparent altitude observed on its surface the horizontal parallax being given. Note. The moon's horizontal parallax is found in the Nautical Almanac for that year.

Enter the table with the moon's horizontal parallax at the top, and (under it) against the observed alt. you will have the parallax of alt. For the odd minutes of alt. and of horizontal parallax, you may readily obtain the proportion for the quantity corresponding them. As for ex. the 20th Sept. 1772.

At 8 h. 50' A.M. the moon's apparent altitude was $43^{\circ} 39' 30''$

By Ephemeris the moon's horizontal parallax was $56' 25''$ meridian of Greenwich 9 A.M.

By Table under $56'$, and against 43° altitude is $40' 57''$

And against 44° altitude is $40' 17''$

Difference of altitude 1° gives $0. 40$ decreasing.

And if $60'$ of altitude decrease the parallax of altitude $40''$, then $39\frac{1}{2}$ will decrease it 27. Hence from $40' 57''$ take $27''$ and there will remain $40' 30''$ for the parallax of altitude with $56'$ horizontal parallax; but $1'$ or $60''$ increase of horizontal parallax, will increase parallax of altitude $43''$ by the table as is under, difference in the right hand column.

Hence as $60'' : 43'' :: 25'' : 18''$ the increase $\therefore$ to $40' 30''$
+ add 18

The parallax of the moon at $43^{\circ} 39\frac{1}{2}$ apparent altitude is $40' 48''$

Refraction, by table, page 13 of the altitude is $1' 00''$

The difference between the parallax and the refraction is $39' 48''$
add $43' 39' 30''$

The true altitude of the moon, } is $44' 19' 18''$

above the rational horizon, } and $45^{\circ} 40' 42''$ is the true Z. distance

And because the seconds exceed 30, I take $45^{\circ} 41'$ for the true zenith distance, as the seconds of altitude, are of no consequence.

Remark 1. That the true time of an observation may generally be found from the mean of the times, and of the altitudes of the sun taken at the same time, that the distances of sun and moon were observed; for the distances should be taken when the moon is pretty high or considerably higher than the sun, and with its limb well illuminated, when you can make by means of your dark glasses, the limbs or edges of the sun and moon appear clear and sharp; and free from glare; at such times the limbs are said to be well defined.

2. It frequently happens that you may take the altitude of the sun, and the time, by your watch, when the sun is in a proper position, as directed in p. 9, art. 7, and at 1, 2, or 3 hours before or after, you take the distances of sun and moon, or moon and star. Having found how much your watch was (or is) fast or slow, you may reduce the mean of the times by your watch, when the distances were taken, to the true solar time.

OF REFRACTION,

Or the bending of the Rays of Light, as they pass through the Atmosphere of the Earth; that is, as they pass through that part of the Air, which receives Vapours and Exhalations.

DEF. I. **B**Y the rays of light are to be understood their least parts, and those as well successive in the same lines, as contemporary in several lines. For it is manifest that light consists of parts both successive and contemporary: Rays of light may be considered to be lines reaching from the luminous body to the body illuminated, and the refraction of those rays to be the bending or breaking of those lines in their passing out of one medium into another.

2. By the eclipses of Jupiter's Satellites is proved, that light is propagated in time, and is about seven minutes of time in passing from the sun to the earth.

3. The angle of incidence, is that angle which the line described by the incident ray contains with the perpendicular to the reflecting or refracting surface at the point of incidence.

4. The angle of reflection or refraction, is the angle which the line described by the reflected or refracted ray, makes with the perpendicular, supposed to be drawn, to the reflecting or refracting surface, at the point of incidence, and in uniform mediums it is bent at its entering into the medium, and then proceeds in a right line; but in passing through mediums which increase or decrease in density, the angle of refraction at any point, is the angle contained between the perpendicular drawn from that point, and the ray from thence proceeding; and this angle decreases as the ray passes through increasing mediums; but the angle increases as it passes through decreasing mediums.

5. The sines of incidence, reflection, and refraction, are the sines of the angles of incidence, reflection, and refraction; as the angles ABD, DBC and EBF, in Fig. 1.

A X I O M S.

1. The angles of reflection and refraction, lie in one and the same plane, with the angle of incidence.

2. The angle of reflection is equal to the angle of incidence.

3. If the refracted ray be returned directly back, that is, if it be reflected perpendicularly, it shall be refracted into the line before described by the incident ray, to the point of incidence.

4. Refraction out of the rarer medium into the denser, is made towards the perpendicular, that is, the angle of refraction is less than the angle of incidence; but refraction out of a denser into a rarer medium, is made from the perpendicular, Def. 4. By Fig. 1, AD is the sine of incidence = DC the sine of reflection, and EF is the sine of refraction.

If the proportion or ratio of the sine of incidence to that of refraction, be known, in any one inclination of the incident ray, it will also be known in all other inclinations: Thus if the refraction be made out of air into water, the sine of incidence of the red light, is to the sine of its refraction into the water as 4 to 3. Thus, $AD = aG$, and $aG : FE :: 4 : 3$. If out of air into glass, the sines are as 17 to 11. In rays of light, of other colours, the sines have other proportions; but the difference is so little that it needs seldom to be considered. [See Sir Isaac Newton's Optics].

5. A ray of light in passing through a medium which increases in density, makes the ray bend toward the denser parts continually as it passes through it; by which all rays will appear as if they had proceeded from other points than they really did. The sun, moon and stars, and all objects on the earth, appear higher by refraction, than they are; the angle or quantity of refraction, at different apparent altitudes, in a middle state of the air, is in the table of refraction.

Fig. 4. Suppose BED to represent the surface of the hemisphere of the earth, and ALGH the upper surface of the atmosphere; bp , a ray of light which enters into the atmosphere at p , and passes to F, in the curve continually bending towards the perpendicular pC , and at F, it is

is the deepest immersed; from F it proceeds to L; where it quits the air, and is refracted from the perpendicular LO, into the right line LM; and the power of refraction at L, will be double to the refraction at F, because the ray in its passage from *p* to F, passed through the like medium as it did from F to L, and in course, the effects will be equal.

Suppose a ray of light proceeding from the upper edge of the sun at *a*, enters the atmosphere at I, and is refracted from its direction Ir, to E, the place of an observer, who will see the sun's upper edge in direction of the line of the sensible horizon Ed, in which it entered the eye. the horizontal refraction is the greatest, and it varies from 29 to 37 minutes in the latitude of London; consequently when the sun appears in the horizon*, he is 33 minutes below it by a mean refraction; and in greater latitudes something more, the atmosphere being more dense. Let *m* represent the place of an observer on the top of a mountain, *ml*, his visible, and *mn*, his true horizons, the visible horizon at *s*, will appear higher than it is, occasioned by the refraction of the rays coming from thence to the eye, from a denser to a rarer medium; for by Axiom 5, and Def. 4, the perpendicular to the refracting surface is Se, and the point of incidence is at S, which point will appear to be lifted up, by the refraction of the ray Sm, it being bent from the perpendicular†; this refraction added to the apparent horizontal refraction of the sun or stars, gives the true refraction as determined by calculations. When it is lessened by the sun's horizontal parallax, the mean parallax of the sun being 8,75 seconds, as determined from the observations of the transits of Venus over the sun in the years 1761 and 1769. The horizontal refractions must be the greatest, because the rays in that position pass through the greatest space of the atmosphere, also through the greatest quantity of vapour. The angle *nm*, is the depression of the visible horizon, below the true horizon; and the rational horizon is that which passes through the center of the earth, as AH, respecting E, parallel to the true horizon *bd*.

Suppose an observer, on shore, with an astronomical quadrant, observes the lower edge of the sun's limb to be 40'.20" above the surface of the sea; the altitude of the telescope 20 feet, higher than the sea; when the diameter of the sun is 32'.30", and his horizontal parallax 9", the horizontal refraction 33'; to find the true zenith distance of the sun. Thus, the apparent alt. above the visible horizon 40'.20" | L.L. + 16'.15" — 26'.57" ref.
The difference between the quantities 14.49 | H.P. + 0.9 — 4.16 for 20 feet high.
True alt. of sun's center above the hor. 25.31 + 16.24 — 31.13
Z. dist. 64.29 their diff. is 14.49

Art. 1. The rays of light which fall obliquely on a medium, in passing through it, if the medium be uniform, are only bent at their entering into, and coming out of it; but in passing through mediums, which gradually increase or decrease (as the atmosphere) the rays at entering in, and passing through it, are gradually bent, so long as the medium increases through which they pass; and they will enter the eye, refracted the same quantity, as if the medium had been uniform, and of the same density as that where they enter the eye.

Remark 1. The quantity of refraction is the same, whether the rays pass through a mile or an inch of it, in the same medium, with that near the eye. And because the last inch of atmosphere, before the ray enters the eye, gives the quantity of its refraction; therefore by passing through the whole atmosphere the quantity of refr. is not increased. For example;

Suppose the atmosphere of the earth (through which an oblique ray passes to the eye) to be divided into three mediums; the first medium the ray passes through, suppose it to be refracted 1, the second medium double in density to the first; and the third medium treble to the first; then I say the refraction in passing through the first being one, in passing through the second medium it will be refracted $1 + 1 = 2$ by both these mediums; and in passing through the third medium it will be refracted $1 + 1 + 1 = 3$, by all the three mediums: because the succeeding mediums have a refractive power in proportion to their density. Q.E.D.

Remark 2. Since the rays of light are the most refracted in that part of the atmosphere which is the most dense, it may happen in many circumstances, that the rays may be reverted before they come to the eye; for instance, if the air round a telescope, through which a person is observing the altitude of a star, be rarer than it is at some distance from the telescope;

* Note. The true horizon of any place, is a tangent to the curve of the earth's surface at that place; which is 90 deg. from the zenith in every position and place.

† Towards the denser parts of the medium.

by the foregoing principles, the curve of the rays will be inflected before they enter the Telescope, and in course the altitude of the star will appear less than it would have done, provided the medium within and without the place of observation, had been the same. For this reason, and the methods used in many observatories; it is evident, that no great accuracy can possibly be drawn from their observations, save the meridian ones.

N. B. That Observatories which are built on the summits of hills, and those in valleys, are both liable to many inconveniences, which those that stand on spacious planes are not subject to.

Sir Is. Newton's Optics, Prop. 7. The perfection of telescopes is impeded by the different refrangibility of the rays of light. The diameter of the image made by the rays passing through a convex glass. See p. 73, 74. Also, Part I, B. 1, p. 8. If the theory of making telescopes could at length be fully brought into practice, yet there would be certain bounds beyond which they could not perform: For the air through which we look at the stars, is in a perpetual tremor; as may be seen by the shadows of high towers; and by the twinkling of the fixed stars. But these stars do not twinkle when viewed through telescopes which have large Apertures. For the rays of light which pass through divers parts of the Aperture, tremble each of them apart; and by means of their various, and sometimes contrary tremors, fall at one and the same time upon different points in the bottom of the eye, and their trembling motions are too quick and confused to be perceived severally.

And all these illuminated points, constitute one broad lucid point, and cause the star to appear broader than it is:

In fig. 5. Let r, r, r , be rays of light, which enter the denser part of the atmosphere at 1, 2, 3, through which they pass to the points 5, 6, 7, where the ray at 5 enters the glass medium, and is refracted towards the perpendicular $5b$, till it arrive at 8, where it enters the water medium, and is there refracted towards the perpendicular be , at 9 it enters the air, and is refracted from the perpendicular $9f$, into the curve 910 ; the other rays $r2, r3$, will be bent and refracted, as appears in the figure, by principles before mentioned.

In this, the body fgb , represents a square prism of glass, put into a vessel of water of a similar form.

The ray $r3$, enters the atmosphere perpendicularly, and therefore is not bent till it enters the glass vessel, which holds the water at 7, and when it enters the water it will be bent the contrary way, and then pass in a right line to t , where it will be bent the same way as it was at 7; and if it be there refracted, so as to pass in a line perpendicular to the surface of a prism, where it entered; it will proceed in that right line through the water and the side of the glass at v , and at entering the air it will be bent towards the greatest body of the atmosphere into the curve vu , and the like of any other. The angle of incidence of a ray being given at its entering into any of the mediums; its inclination may be determined at any point, in its passage through the different mediums of glass and water. The sine of incidence to that of refraction out of air into water, being as 4 : 3, and out of air into common glass as 17 : 11, and out of air into crystal as 25 : 16, which reduced to the ratio of unity will be, the

	Sine of incidence,		Sine of refraction
Rays of light,	out of air into water,	as 1	to 0,75000
————	out of air into plate glass	1	to 0,64700
————	out of air into flint glass	1	to 0,64516
————	out of air into rock crystal	1	to 0,4600
————	out of water into flint glass	1	to 0,86020

If Light pass through many refractive Mediums gradually denser and denser, which terminate with parallel surfaces, the sum of all the refractions will be equal to the single refraction, which it would have suffered in passing immediately out of the first medium into the last. Newton's Optics, book 2, part 3, prop. 10.

The parallaxes of altitude, are to each other as the co-sines of the apparent altitudes. To find the parallax of alt. you must get the horizontal parallax at that time, and then say, as $R : \text{hor. par.} :: \text{cs. appt. alt.} : \text{par. of alt.}$. Suppose the moon's hor. par. $53'$, appt. alt. 30° ,
Then Logistical log. of $53' 00'' = 10,5310$ Rad. added
Sub. $\text{cs. of appt. alt. } 30' = 9,9375$

L. L. of Parx. of alt. $45' 54'' = 0,5935$ by this method the following table was computed.

A TABLE

A TABLE shewing the Distance of the Visible Horizon, for different Heights of the Observer's Eye above the Sea, together with the True and apparent Depreffion of the Visible Horizon. [See Fig. 3. Plate 1].

Height of the eye in feet.	Dist. of the hor. in miles.	Depress. of the horizon.	Refrac. tion of the ho- rizon.	Apparent dip of the horizon.
1	1,25	1' 00''	0' 8''	0' 52''
2	1,75	1 30	0 10	1 20
3	2,12	1 50	0 11	1 39
4	2,5	2 22	0 13	2 09
5	2,75	2 20	0 14	2 16
6	3,0	2 36	0 15 $\frac{1}{2}$	2 20 $\frac{1}{2}$
7	3,25	2 50	0 17	2 33
8	3,5	3 2	0 18	2 44
9	3,7	3 16	0 19	2 57
10	3,85	3 32	0 20	3 12
12	4,25	3 40	0 22	3 18
14	4,7	4 01	0 24	3 37
16	5,0	4 19	0 26	3 53
18	5,25	4 32	0 27 $\frac{1}{2}$	4 4
20	5,5	4 45	0 28 $\frac{1}{2}$	4 16 $\frac{1}{2}$
24	6,0	5 12	0 31	4 41
28	6,5	5 37	0 34	5 3
32	7,0	5 55	0 36	5 19
36	7,4	6 10	0 38	5 32
40	7,8	6 30	0 40	5 50
42	8,0	6 48	0 42	6 6
48	8,5	7 21	0 44	6 39
54	9,0	7 48	0 47	7 1
59 $\frac{1}{2}$	9,5	8 13	0 49	7 24
66 $\frac{1}{2}$	10,0	8 38	0 52	7 46
95	12,0	10 23	1 2	9 21
130	14,0	12 6	1 12 $\frac{1}{2}$	10 53 $\frac{1}{2}$
147	15,0	12 58	1 18	11 40
168	16,0	13 50	1 23	12 27
215	18,0	15 34	1 33 $\frac{1}{2}$	14 00 $\frac{1}{2}$
266	20,0	17 18	1 44	15 34
336	24,0	20 45	2 4 $\frac{1}{2}$	18 40 $\frac{1}{2}$
600	30,0	26 00	2 36	23 24
yards.	miles.	' ''	' ''	' ''
400	41,5	0 36 45	3 40	33 05
738	58,0	0 50 00	5 00	45 00
800	60,14	0 52 00	5 12	46 48
1200	73,6	1° 3' 40''	6 22	56 18
1600	85,0	1 13 30	7 21	1° 6' 9''
1 mile.	89,24	1 17 10	7 43	1 9 27
miles.				
1 $\frac{1}{4}$	99,75	1 26 15	8 37	1 17 38
1 $\frac{1}{2}$	109,3	1 34 30	9 27	1 25 3
1 $\frac{3}{4}$	118,0	1 42 00	10 12	1 31 48
2	126,1	1 49 00	10 54	1 38 6
3	154,4	2 13 30	13 21	2 00 9

The $\angle$ of depression bAE , is $= \angle ACE$, because the $\angle CAE$ is the comp. to each of them, as by the Fig. is evident.

Let C , represent the center of the earth, A , the summit of a place on its surface; $gfaE$, a part of a great circle of the earth; EA , a tangent to it, drawn from the point of contact E ; also the tangent line ab , and its parallel Ab , may represent the true horizontal directions of the different altitudes of the observer above the horizon, or surface of the sea; in order to determine the inclination (or depression) of the visible horizon, below the true, at different altitudes of the observer; we have given the semidiameter of the earth $= 3975$ miles, and the height of the observer's eye (by Fig. 3) in the triangle, AEC ; Let Aa , be $= 400$ yards. Then $\sqrt{AC^2 - EC^2} = AE$, the distance of the horizon from A . and since the $\angle E$, is a right angle, and therefore $=$ the $\angle A + C$; but the $\angle C = \angle bAE$ of depression. Hence $AC : CE :: R : cs.$ of the $\angle ACE$ of depression.

Now $3975 \times 1760 = 6996000$ yards $= Ca = CE$, and $AC = 6996400$ yards.

By Trig. yds.

	parts of a mile in yds.
As 6996400—6,8448746	$\frac{1}{8}$ 220
: 6996000—6,8448498	$\frac{2}{8}$ 440
$:: R : cs. \angle C - 36' 45'' - 9,9999752$	$\frac{3}{8}$ 660
	$\frac{4}{8}$ 880
And $R : 6996400 - 6,8448746$	$\frac{5}{8}$ 1100
$:: S 36' 45'' - 8,0289740$	$\frac{6}{8}$ 1320
	$\frac{7}{8}$ 1540
: 73088,5—4,8638486	1 1760

yds. miles.

Now $73088 = 41,5$ the distance of the horizon.

Ex. 2. To find the horizontal dip and dist. of the horizon, the observer being one mile high.

1. As the earth's rad. $+ 1$ mile $= 3976 - 3,5994464$
: ac. the rad. in miles $= 3975 - 3,5993371$

$:: R : cs. \text{ of the dip } - 1^\circ 17' 10'' - 9,9998907$

2. $R : \text{ the hyp. } CA, \left. \begin{array}{l} \\ \end{array} \right\} 1^\circ 17' 10'' - 8,3511194$
 $:: s, \text{ of dip the } \angle C$

: dist. of the vis. hor. $AE - 89,24 - 1,9505658$

To find Aa . As $cs. \text{ of the } \angle \text{ of depref. } : R :: CE$
: CA and $CA - CE = Aa$.

Note, that if any number in the first column be multiplied by 4, and its corresponding number in the 2d col. by 2, the products will shew the alt. of the observer, and the dist. of the vis. hor. Since the rays of the hor. are refracted (as in the 4th col.) that quantity must be taken from the angle of the dip, obtained by calculation.

The

The variation of the horizontal refractions are from 29' to 37', in the lat. of London (and at the Royal Observatory at Greenwich) and considerably greater to the Northward: These being the rarest and densest atmospheres at the Royal Observatory, I may therefore infer the mean refraction near the banks of the Thames, to be about 34'.

A New TABLE of REFRACTIONS, adapted to the middle State of the Air.

Ap. Alt.	Refract.	Ap. Alt.	Refract.	Ap. Alt.	Refract.	Ap. Alt.	Refract.
0	0 33 0,0	6	50 7 30,2	19	30 2 39,4	62	0 0 30,4
0	5 32 10,4	7	0 7 20,5	20	0 2 35,1	63	29,1
0	10 31 22,2	7	10 7 11,1	20	30 2 31,0	64	27,8
0	15 30 35,4	7	20 7 2,1	21	0 2 27,2	65	26,5
0	20 29 49,7	7	30 6 53,4	21	30 2 23,6	66	25,3
0	30 28 22,3	7	40 6 45,1	22	0 2 20,3	67	24,1
0	32 28 4,8	7	50 6 37,1	23	2 13,7	68	22,9
0	36 27 30,3	8	0 6 29,4	24	2 7,4	69	21,7
0	40 26 59,7	8	10 6 22,0	25	2 1,6	70	20,6
0	50 25 41,8	8	20 6 14,8	26	1 56,2	71	19,5
1	0 24 28,6	8	30 6 8,0	27	1 51,2	72	18,4
1	10 23 19,8	8	40 6 1,3	28	1 46,6	73	17,3
1	20 22 15,2	8	50 5 54,8	29	1 42,4	74	16,2
1	30 21 14,7	9	0 5 48,5	30	1 38,4	75	15,1
1	40 20 17,9	9	10 5 42,4	31	1 34,6	76	14,0
1	50 19 24,8	9	20 5 36,5	32	1 31,0	77	13,0
2	0 18 35,0	9	30 5 30,9	33	1 27,6	78	12,0
2	10 17 48,4	9	40 5 25,4	34	1 24,4	79	11,0
2	20 17 4,5	9	50 5 20,0	35	1 21,4	80	10,0
2	30 16 23,8	10	0 5 14,8	36	1 18,5	81	9,0
2	40 15 45,4	10	15 5 7,3	37	1 15,7	82	8,0
2	50 15 9,4	10	30 5 0,1	38	1 13,0	83	7,0
3	0 14 35,6	10	45 4 53,2	39	1 10,4	84	6,0
3	10 14 3,9	11	0 4 46,6	40	1 7,9	85	5,0
3	20 13 34,1	11	15 4 40,3	41	1 5,5	86	4,0
3	30 13 6,2	11	30 4 34,3	42	1 3,3	87	3,0
3	40 12 39,6	11	45 4 28,6	43	1 1,1	88	2,0
3	50 12 14,7	12	0 4 23,2	44	59,0	89	1,0
4	0 11 51,1	12	20 4 16,1	45	57,0	90	0,0
4	10 11 28,9	12	40 4 9,4	46	55,0		
4	20 11 7,9	13	0 4 3,0	47	53,1	Height of Eye.	Ap Dip
4	30 10 48,0	13	20 3 56,9	48	51,2	Feet.	Min.
4	40 10 29,2	13	40 3 51,1	49	49,4	4	1,9
4	50 10 11,3	14	0 3 45,5	50	47,6	6	2,3
5	0 9 54,3	14	20 3 40,1	51	45,9	8	2,7
5	10 9 38,2	14	40 3 34,9	52	44,2	10	3,0
5	20 9 22,8	15	0 3 29,9	53	42,6	12	3,3
5	30 9 8,0	15	30 3 23,7	54	41,0	14	3,6
5	40 8 54,0	16	0 3 16,9	55	39,6	16	3,9
5	50 8 40,6	16	30 3 10,5	56	38,2	18	4,1
6	0 8 27,8	17	0 3 4,5	57	36,7	20	4,3
6	10 8 14,9	17	30 2 58,9	58	35,5	24	4,7
6	20 8 2,8	18	0 2 53,6	59	34,2	28	5,0
6	30 7 51,1	18	30 2 48,6	60	33,0	34	5,5
6	40 7 40,3	19	0 2 43,9	61	31,7	40	6,0

TABLE II.

A Table of the Minutes to be applied to the observed Alt. of the lower Limb of the Sun.

App. altitude of lower edge of the sun, the horizontal dip. being 4 minutes.

Min. subtr. or add. fr. or to the ap. alt. of lower edge of the sun obs.

from 0 to 5 Subt. 21
5 to 10 ditto 20
10 to 16 ditto 19
16 to 22 ditto 18
22 to 28 ditto 17
28 to 36 ditto 16
36 to 44 ditto 15
44 to 52 ditto 14
52 to 60 ditto 13

fr. 1 to 1 8 Subt. 12
to 1 16 ditto 11
to 1 26 ditto 10
to 1 36 ditto 9
to 1 48 ditto 8
to 2 0 ditto 7
to 2 13 ditto 6
to 2 28 ditto 5
to 2 45 ditto 4
to 3 2 ditto 3
to 3 20 ditto 2
to 3 42 ditto 1
to 4 10 — 0
to 4 40 Add 1
to 5 16 ditto 2
to 5 56 ditto 3
to 6 48 ditto 4
to 8 0 ditto 5
to 9 30 ditto 6
to 11 40 ditto 7
to 15 ditto 8
to 20 ditto 9
to 30 ditto 10
to 50 ditto 11
to 90 ditto 12

Of the PARALLAXES of the MOON, or the Difference between the true and apparent Place at different Altitudes.

In fig 2, C represents the centre of the earth; TAB places on it's surface; DEF the orb of the moon; GHI the orb of Venus; KLM that of Mars; NOP that of Jupiter; and QRZ that of Saturn, or the fixed stars,

Let A be the place of an observer; AR his horizon; Z the Zenith; D the Moon in the rational, and E her place in the true horizon—It is evident by the Fig. that the moon's parallax will be the greatest, when she is in the apparent horizon at E; for the angle AEC is greater than the angle ADC; because AE and EC, are less than AD and CD, or any other two lines, from A and C meeting in the vertical arch DE (AC being constant). Consequently the quantity of the angles AEC, ADC, will be as the radius to the co-sines of the apparent altitudes, or as the sines of the zenith distances.

The like of the planets; the fixed stars being at such immense distances, make their diurnal parallaxes not sensible, and their annual parallaxes so little, as not to be determinable by observations, except a few of them.

Remark 1. The nearer a planet or star is to the earth, the greater is its parallax

2. All parallaxes of altitude vanish in the zenith.

3. The parallax causes the moon and planets to appear in the heavens, in different places than those they really are; and in the meridian, the planet will have parallax in altitude, longitude, latitude, and decl. only; in all other positions, there will be a parallax in alt. long. lat. decl. and right ascen. [See the Sea Officer's Companion, p. 27, 28. Also Keill's Astronomy, Lect. 14, p. 150; and De la Caille's Elements of Astronomy, ch. 3; or Ferguson's Astronomy, p. 100, ch. 9.

4. The true place of the moon, when at E, is in the line CE, continued to the sphere of the fixed stars, as at *t*, and its apparent place to a Spectator at A is at R. When the moon is at *a*, the true place is at *f*, and apparent at *c*; when the moon is at *b*, the true place is at *g*, and apparent at *e*, which continually decreases, as the altitude increases.

5. When the moon is in perigee, the horizontal parallax at the equator is $61' 38''$; when in apogee, $53' 10''$.

But the equatorial parallax is greater, than it is in any place to the North or South of it; by reason of the diameter of the earth at the equator, being greater than at any other place; and therefore a greater side than AC, will subtend a greater angle at E, all other circumstances being the same.

6. The parallax of the moon or star does not alter its true vertical position.

7. The parallax makes the moon and planets appear further from the meridian than they are: Consequently the Longitude will be apparently increased in the Eastern, but decreased in the Western hemispheres.

REMARKS of REFRACTION, out of WATER into AIR.

Fig. 1. Let BH $\overline{mn}$ represent a vessel, fill'd with water; *bn* a straight rod of any kind, by the power of refraction all the different parts from *p* to *n*, will appear, as if they were lifted up, and the part *n* will appear at *o*; and therefore the rod will appear to be bent at the point *p*, in the surface of the water, from it's true direction *bn*, into the line RO: Def. 4. And by the same cause, if a piece of money, or any other thing be put at *n*, and you to stand so as just to see one edge of it, when there is no water in the vessel; and then pour water into it, and you may see more and more of the piece, as the depth of water increases upon it; till you see the whole of it.

Remark 2. That the visibility of the bottom of the vessel, or of the line *nm*, increases directly as the depth of the water increases; thus, if *nt* be equal one inch, and NB equal to 10 inches, then each inch depth of water, would give one-tenth of an inch from *n* towards *t*; and 10 depth would give *nt*, which would appear at *ro*.

3. Why does the increased depth of water, make this increase of visibility, since the angle of refraction is the same, at all depths of the water, when the thing is covered? Answer: by reason

reason that the bend of refraction is made at a greater distance from the object, at the bottom of the vessel; when it is full of water, or the like liquid; when the parts at n and t , will appear at o and r , and the eye at b , would see the whole diameter nt ; the angle of refraction being Rpb , by axiom 4; and the like of any other.

6. By reason of the moon being much nearer to the earth, than any of the planets, it therefore has a much greater parallax at its rising and setting; the quantity of the horizontal parallax of the moon is to be found in the Nautical Almanac of the given year, under the month, and against the day, in the columns, intitled,

Hor. Par.	Hor. Par.	in minutes and seconds, (M. S.); and by the difference between
moon at	moon at	the quantities of the parallaxes at noon and midnight, the paral-
noon.	midnight.	lax may be found at any time.

Ex. To find the horizontal parallax. the 18 of September, 1776, at 5 h. in the afternoon.

H. P. at noon - $55'.18''$ | diff. $17''$, the increase in 12 h. which may be called $1''.\frac{1}{2}$,

at midnight 55.35 | an hour increase; hence the hor. parx. at 5 P. M. is $55'.26''$.

And the like of the semidiameter of the moon, which is at 5 P. M. $15'.16$.

Also, since the moon (as well as a star) is at the greatest distance from the observer, when it is in the horizon; and as it approaches the zenith, its diameter appears to be increased, and when it is in the zenith, the diameter will appear to be the greatest in that day's revolution; for when any planet is in the zenith of an observer, he is nearer to it, by a semidiameter of the earth, which is near 4000 English miles. This is evidently shewn in Fig. 2, in the plate to the Appendix.

A TABLE of the apparent Increase of the Semidiameter of the Moon at different Altitudes, to be added to Hor. Diam.

Moon's app. Alt.	Moon's Hor. Parlx.		Moon's Alt.	Hor. Parallax.	
	54'	52'		54'	62'
0	"	"	0	"	"
5	1	1	40	9	12
8	2	$2\frac{1}{2}$	44	10	13
12	3	4	48	10	14
16	4	5	52	11	15
20	5	6	56	12	16
24	6	7	65	13	17
26	6	8	74	14	18
30	7	9	85	15	19
34	8	10	90	15	20

• 3. When you take the distance of the moon and star, at some time after midnight, as at 2, 3, 4 or 5, A.M. and think it dubious to trust to the going of the watch, from the time it was last corrected by the sun; in such case, defer computing the observation of longitude till you have got an observation of the sun's altitude for finding the time that the watch is fast or slow, and then correct the time of the longitude observation; as in the following examples.

4. If the distance of the moon from the sun, or a star, be observed, when they are near the same azimuth; their altitudes must be well determined, as the reduced distance will be greatly affected thereby.

5. If The objects happen to be observed in opposite azimuths, at which time the angle at the zenith vanishes, then the rule will be reduced to the following one; which is this: The sum of the true zenith distances is the true reduced distance. I once made an observation of the moon and star in opposite azimuths, when the arch of their dist. passed through the zenith.

If they be in the same vertical, the diff. of their true zenith distances is their reduced dist. By observations of this kind, the longitude of places may be well determined.

* That is when their distances does not exceed 30 degrees

A TABLE of the MOON'S PARALLAX of ALTITUDES.

Horizontal Parallax of the Moon.																			
Moon's ap. alt. °	53°		54°		55°		56°		57°		58°		59°		60°		61°		Dif.
°	'	"	'	"	'	"	'	"	'	"	'	"	'	"	'	"	'	"	'
1	52	59	53	59	54	59	55	59	56	59	57	59	58	59	59	59	60	59	60
2	52	57	53	57	54	57	55	57	56	57	57	57	58	57	59	57	60	57	60
3	52	55	53	55	54	55	55	55	56	55	57	55	58	55	59	55	60	55	60
4	52	52	53	52	54	52	55	51	56	51	57	51	58	51	59	51	60	51	60
5	52	48	53	47	54	47	55	47	56	47	57	47	58	47	59	46	60	46	60
6	52	43	53	42	54	42	55	41	56	41	57	41	58	41	59	40	60	40	60
7	52	36	53	36	54	35	55	35	56	34	57	34	58	34	59	33	60	33	60
8	52	29	53	28	54	27	55	27	56	26	57	26	58	25	59	25	60	24	59
9	52	21	53	20	54	19	55	19	56	18	57	17	58	17	59	16	60	15	59
10	52	12	53	11	54	10	55	9	56	8	57	7	58	6	59	5	60	4	59
11	52	2	53	0	53	59	54	58	55	57	56	56	57	55	58	54	59	53	59
12	51	51	52	49	53	47	54	46	55	45	56	43	57	42	58	41	59	39	58
13	51	39	52	37	53	35	54	34	55	32	56	31	57	29	58	27	59	26	58
14	51	25	52	23	53	21	54	20	55	18	56	16	57	14	58	12	59	10	58
15	51	12	52	9	53	7	54	4	55	3	56	1	56	59	57	57	58	55	58
16	50	57	51	54	52	53	53	50	54	47	55	45	56	42	57	40	58	33	57
17	50	41	51	39	52	36	53	33	54	30	55	28	56	25	57	22	58	20	57
18	50	24	51	21	52	18	53	15	54	12	55	9	56	6	57	3	58	0	57
19	50	7	51	4	52	0	52	57	53	53	54	50	55	47	56	44	57	40	57
20	49	48	50	45	51	41	52	37	53	33	54	30	55	26	56	23	57	19	56
21	49	29	50	25	51	21	52	17	53	13	54	9	55	5	56	1	56	57	56
22	49	9	50	5	51	0	51	56	52	51	53	47	54	42	55	38	56	33	56
23	48	47	49	42	50	36	51	33	52	29	53	24	54	19	55	10	56	9	56
24	48	25	49	20	50	14	51	9	52	4	52	59	53	54	54	49	55	43	55
25	48	2	48	56	49	50	50	44	51	39	52	34	53	28	54	23	55	17	55
26	47	38	48	33	49	26	50	19	51	13	52	7	53	1	53	55	54	49	54
27	47	13	48	7	49	0	49	54	50	47	51	41	52	34	53	27	54	21	54
28	46	47	47	40	48	33	49	26	50	19	51	12	52	5	52	58	53	51	53
29	46	21	47	14	48	6	48	58	49	51	50	44	51	36	52	28	53	21	52
30	45	54	46	46	47	38	48	30	49	22	50	14	51	6	51	57	52	49	52
31	45	26	46	17	47	8	47	59	48	51	49	42	50	34	51	25	52	17	51
32	44	57	45	47	46	38	47	29	48	20	49	11	50	2	50	53	51	44	51
33	44	27	45	17	46	7	46	57	47	48	48	38	49	28	50	18	51	9	50
34	43	57	44	46	45	36	46	25	47	15	48	5	48	54	49	44	50	34	50
35	43	25	44	14	45	3	45	52	46	41	47	30	48	19	49	8	49	58	49
36	42	52	43	41	44	30	45	18	46	7	46	56	47	44	48	33	49	21	49
37	42	19	43	7	43	55	44	43	45	31	46	19	47	7	47	55	48	43	48
38	41	46	42	33	43	20	44	7	44	55	45	42	46	29	47	16	48	4	47
39	41	11	41	58	42	44	43	31	44	17	45	4	45	51	46	37	47	24	47
40	40	36	41	22	42	8	42	54	43	40	44	26	45	12	45	58	46	44	46
41	40	0	40	45	41	30	42	16	43	2	43	46	44	31	45	16	46	2	46
42	39	23	40	8	40	52	41	37	42	21	43	6	43	51	44	35	45	20	45
43	38	46	39	20	40	13	40	57	41	41	42	25	43	9	43	53	44	37	44
44	38	8	38	51	39	34	40	17	41	0	41	43	42	26	43	10	43	53	43
45	37	29	38	11	38	54	39	37	40	18	41	0	41	43	42	25	42	8	42

A TABLE

A TABLE of the MOON'S PARALLAX of ALTITUDES.

Horizontal Parallax of the Moon.																				
Moon's ap. alt. °	53'		54'		55'		56'		57'		58'		59'		60'		61'		Dit. 60"	
°	'	"	'	"	'	"	'	"	'	"	'	"	'	"	'	"	'	"	"	
46	36	49	37	30	38	12	38	54	39	35	40	17	40	59	41	41	42	22	42	
47	36	09	36	49	37	30	38	11	38	52	39	33	40	14	40	55	41	36	41	
48	35	28	36	08	36	48	37	28	38	08	38	48	39	28	40	08	40	49	40	
49	34	46	35	25	36	05	36	44	37	23	38	02	38	42	39	21	40	01	39	
50	34	04	34	42	35	21	35	59	36	38	37	17	37	55	38	34	39	13	39	
51	33	21	33	59	34	37	35	14	35	52	36	30	37	07	37	45	38	23	38	
52	32	38	33	16	33	52	34	29	35	05	35	42	39	19	36	56	37	33	37	
53	31	54	32	30	33	06	33	42	34	18	34	54	35	30	36	06	36	42	36	
54	31	09	31	44	32	19	32	55	33	30	34	05	34	41	35	16	35	51	35	
55	30	24	30	58	31	32	32	07	32	41	33	16	33	50	34	25	34	59	34	
56	29	38	30	11	30	45	31	19	31	52	32	26	32	59	33	33	34	06	34	
57	28	52	29	24	29	57	30	30	31	02	31	35	32	08	32	41	33	13	33	
58	28	05	28	37	29	08	29	40	30	12	30	44	31	16	31	48	32	19	32	
59	27	18	27	48	28	19	28	50	29	21	29	52	30	23	30	54	31	25	31	
60	26	30	26	59	27	30	27	59	28	30	28	59	29	30	30	00	30	30	30	
61	25	42	26	11	26	40	27	09	27	38	28	07	28	36	29	05	29	34	29	
62	24	53	25	21	25	49	26	17	26	45	27	13	27	41	28	10	28	38	28	
63	24	04	24	31	24	58	25	25	25	52	26	20	26	47	27	14	27	41	27	
64	23	14	23	40	24	06	24	33	24	59	25	25	25	52	26	18	26	44	26	
65	22	24	22	49	23	14	23	40	24	05	24	30	24	56	25	21	25	46	25	
66	21	33	21	57	22	22	22	47	23	11	23	35	23	59	24	24	24	48	24	
67	20	42	21	05	21	29	21	53	22	16	22	39	23	03	23	26	23	50	23	
68	19	51	20	13	20	36	20	58	21	21	21	43	22	06	22	29	22	51	22	
69	19	00	19	21	19	43	20	04	20	26	20	47	21	08	21	30	21	51	21	
70	18	08	18	28	18	49	19	09	19	30	19	50	20	12	20	31	20	51	20	
71	17	16	17	35	17	54	18	14	18	34	18	53	19	13	19	32	19	51	19	
72	16	23	16	41	16	59	17	18	17	37	17	55	18	14	18	33	18	51	18	
73	15	30	15	48	16	05	16	23	16	40	16	58	17	15	17	33	17	50	17	
74	14	37	14	54	15	10	15	27	15	43	15	59	16	16	16	33	16	49	16	
75	13	43	13	59	14	14	14	30	14	46	15	01	15	16	15	32	15	48	16	
76	12	49	13	04	13	19	13	33	13	48	14	02	14	16	14	31	14	46	15	
77	11	55	12	09	12	23	12	36	12	50	13	03	13	16	13	30	13	44	14	
78	11	01	11	14	11	26	11	38	11	51	12	03	12	16	12	29	12	41	13	
79	10	07	10	19	10	30	10	41	10	53	11	04	11	15	11	27	11	39	12	
80	9	12	9	23	9	34	9	44	9	54	10	04	10	15	10	26	10	36	11	
81	8	18	8	28	8	37	8	46	8	55	9	04	9	14	9	24	9	34	10	
82	7	23	7	32	7	40	7	48	7	56	8	04	8	13	8	22	8	30	9	
83	6	29	6	36	6	43	6	50	6	57	7	04	7	12	7	20	7	27	8	
84	5	34	5	40	5	46	5	52	5	58	6	04	6	10	6	17	6	23	7	
85	4	39	4	44	4	49	4	54	4	59	5	04	5	09	5	15	5	20	6	
86	3	43	3	47	3	51	3	55	3	59	4	03	4	08	4	12	4	16	5	
87	2	48	2	51	2	54	2	57	3	00	3	03	3	06	3	09	3	13	3	
88	1	53	1	55	1	56	1	58	2	00	2	02	2	04	2	06	2	08	2	
89	0	59	0	59	0	59	0	59	1	00	1	01	1	02	1	03	1	04	1	
90	0	00	0	00	0	00	0	00	0	00	0	00	0	00	0	00	0	00	0	

Horizontally.

Horizontally.

E

To

See the Use of this Table in p. 12.

To find the LONGITUDE of a PLACE.

Sept. 18, 1776, at 3h. 7m. by watch, sun's tr. alt. $24^{\circ}.05'$ in lat. $51^{\circ}.29'$ N. dec. $1^{\circ}.31'$ N.
True Z.D. 65.55 co-lat. 38.31 pol. dist. 88.29

To find the TRUE TIME.

Co-lat.	$38^{\circ}.31'$	—	0,205692
Polar dist.	88.29	—	0,000152
Co-alt.	65.55		9,928144
Sum	192.55		9,142205
Half Sum	$96.27\frac{1}{2}$		19,276194
$\frac{1}{2}$ Sum—co-lat.	$57.56\frac{1}{2}$		9,638097
$\frac{1}{2}$ Sum—pol. dist.	$7.58\frac{1}{2}$	s, of $25.45\frac{1}{2}$	

Time by the sun $3h.26'.04'' = 51.31 \angle$ time.
—by the watch $3 \ 7 \ 00$ Moon's hor. parx.
Watch flow $0.19.04$ $55'.24''$

By Rule I. in p. 11.

Appt. dist.	$63^{\circ}.56'$	
Sun's ap. Z.D.	79.30	— 0,0073339
Moon's ap. Z.D.	69.05	— 0,0296063
Sum	212.31	9,6534328
Half sum	$106.15\frac{1}{2}$	9,7812177
Remainders	$26.45\frac{1}{2}$	19,4715907
	$37.10\frac{1}{2}$	9,7357953
		s, of $32^{\circ}.58'.23''$

Its double is the dif. of azimuth $65.56.46$

By Rule II. in p. 11.

R : t. lesser true Z.D. $68^{\circ}.15'.40'' = 10,3993159$
: : cs. dif. of azim. $65.56.46 = 9,6102296$
: t. of 4th arch $45.37.50 = 10,0095455$
Greater true Z.D. $79.35.00$

Rem. 5th arch $33.57.30$

By Rule III. in p. 11.

Co-ar.
Cs. 4th arch $45^{\circ}.37'.50'' = 0,1553475$
: cs. 5th. $33.57.10 = 9,9188154$
: : cs. less true Z.D. $68.15.40 = 9,5686444$
: cs. of red. dist. $63.56.16 = 9,6428073$

As $28'.15'' : 900' :: 70'' = (1'.10''\frac{1}{2}) : 37'$
60 900

1695 - -)63000(37 longitude.

I set my watch to solar time, and made the following observations.

h. m.	sun & moon	Sun's alt.	Moon's alt.
At 4 51	dist. $63^{\circ}.22'$	h. o	alt.
At 4 56 $\frac{1}{2}$	— 63.24	at 5 10.30	$20^{\circ}.55'$
At 5 12 $\frac{1}{2}$	— 63.28	Z.D. 79.30	69.05
3	15.00 — $190^{\circ}.14'$		

At 5.00 mean $63.24.40$

For $\frac{1}{2}$ diam. of sun $+16.00$ } By Nautical
Half diam. of moon $+15.06$ } Almanac.

For moon's alt. $+6.05$ by tab. p. 19.

App. dist. $63.55.51$ of centers.

For which take $63^{\circ}.56'$ the ap. dist.

Ap. Z.D. of moon $69^{\circ}.05'.00''$

Moon's parlx. of alt. $51'.45$ by tab. p. 20.

Ref. subtr. 2.25 by tab. p. 17.

The dif. sub. 49.20 Sun $79^{\circ}.30'$

True Z.D. of moon $68.15.40$ ref. $+5$

True Z.D. of the sun 79.35

Dif. of logs. 3672

$\frac{1}{3}$ subtr. 1224

2448

0711

3159

Note. The calculation here is not made to the greatest accuracy, but to as much as can be required at sea; for observations which are made, only near the truth, cannot require the nicest calculations.

To find the distance by the Nautical Almanac, in p. 106.

Sept. 18th, At 3 h. dist. sun } $62^{\circ}.58'.35''$
and moon - - -

At 6 h. dist. $64.23.21$

In 3 h. increase of dist. $1.24.46$

In 1 h. do. - - $0.28.15$

Dist. by Eph. at 5 h. $63.55.06$

Dist. by obs. reduced $63.56.16$

Dif. 1.10

1st. True

EXAMPLES of various Methods of assuming the Longitude the Ship is in.

Ex. 1st. True time of observation at the ship 5 h. 0'

Assume the place of observation under } 0 0 = 0°. 0' lon.
the meridian of Greenwich

By obs. 0.37 W. By Remark 1, p. 11.

2. Or thus: Tr. time of obs. 5 h. 00 m. at the ship.

Assume Greenwich 1 00 = 15° E. from the ship ∴ ship 15° W.

Time, then at Greenwich 6 00 } Dist. of the sun and moon by Eph. 64°.23'.21"
Dist. deduced from obs. 63.56.16

As 1695" : 900' :: 27'.05" : 14°.23' E. lon. from the place assumed. Dif. 0.27.05

Long. in, by assumption 15' 00" W.

Long. in, by observation 14 23 E. of that assumed.

The Long. in is 0 37 W.

3. True time of obs. 5h.

Assume Greenwich 0 30 = 7° 30' W. from the place of observation.

Time at Greenwich 4 30 by this assumption.

Dist. of Sun and Moon at 4h. 30m. = 63° 40' 58" For at 3h. the dist. is 62° 58' 35"

Reduced dist. by obs. 63 56 16 For 1 30m. add 0 42 23

Dif. 0 15 18 = 918" 4 30 = 63 40 58

As 1695" : 900' :: 918" : 487' = 8° 07' W. by obs. of that assum'd.

Assum'd 7 30 E. Long. in.

Long. in, by observation 0 37 W.

Remark that this method of assuming your long. in, is at your option, and that the same place of the ship will be obtain'd, as in the three assumptions foregoing; for if you assume too much the result will bring you back, but if you assume too little it will carry you on to the place or long. of the ship by observation.

Ex. 2. To find the Lon. of a Place of Observation, when the Dist. observed exceeds 90°.

1772, October the 19th, In the Latitude 51deg. 30m. N. Longitude, assume 5h. = 75 deg. W. where the mean of four distances of the Sun and moon was 89deg. 48m. 10sec. the time 11h. 30m. A. M. at the place of observation.

add 5 00 for W. Longitude = 75° W.

Assumed time at Greenwich 4 30 P. M. of this obs.

Note. The Sun being so near the meridian, the true time could not be determined by the Sun's altitude.

But at 3h. 30m. by watch, the sun's alt. was 14° 40'

For $\frac{1}{2}$ diam. ref. and dip. (by p. 95) + 8 [R. W.'s Navigation.]

True alt. of the Sun. = 14 48 Decl. 10° 24' S.

Z. D. = 75 12 P. dist. 100 24

To find how much the Watch is too fast or slow.

Z. D. 75° 12'

P. D. 100 24 — 0,007194

Co-lat. 38 30 — 0,205850

Sum 214 06 | 9,063724

Half Sum 107 03 | 9,968827

Differences } 6 39 | 19,245595

By Rule, p. 4. } 68 33 | 9,622797 sine of 24° 48' $\frac{1}{2}$

Its double is 49 37 = 3h. 18m. 28sec.

Time at the ship by the watch 3 30 00

Watch fast 0 11 32

Sun's

Sun's app. alt. $27^{\circ} 50'$ ref. 2.	Moon's app. alt. $17^{\circ} 33' 30''$	Parx. of alt. $54' 40''$
app. Z. D. 62 10	app. Z. D. 72 26 30	Ref. subt. 2 50
true Z. D. 62 12	true Z. D. 71 34 40	dif. $51' 50''$

Apparent Dist. of the Limbs observed $89^{\circ} 48' 10''$ as before.

Half diam. of the sun 16 08 } See Nautical Ephem.
 Half diam. of the moon 15 37 }
 For moon's alt. add 5 by table in p. 19.

Apparent distance of centers $90^{\circ} 20' 00''$

Or time at the Ship 11 h. $18\frac{1}{2}$

Time of observed dist. by watch 11 h. 30' Assume the ship 5 $11\frac{1}{2} = 77^{\circ} 52'\frac{1}{2}$ W.
 Subt. watch fast $11\frac{1}{2}$ Time at Greenwich 4 30

True time of obs. at the ship 11 $18\frac{1}{2}$

Assume the dif. of long. 5 $21\frac{1}{2} = 80^{\circ} 22'\frac{1}{2}$ W. The ship off Greenwich.

Assumed time at Greenwich 4 40 P. M.

H. Deg. Min. Sec.

By the Ephemeris. The dist. at 3 P. M. 91 . 9 . 55
 at 6 — 89 . 38 . 38

The dif. in 3h. is 1 . 31 . 17 in 1h. is $30' 26''$
 in 1 $30'$ is 0 . 45 . 39 sub. fr. dist. at 3h.
 in 0 $30'$ is 0 . 15 . 13

True dist. at 4 30 P. M. 90 . 24 . 16 by Eph.

Do. at 4 40 = 90 . 19 . 12

The foregoing particulars are the requisite preparations for all the different methods of calculation.

I. R. WADDINGTON'S Method of computing the Longitude by Trigonometry.

October 19th, App. dist. of the centers $90^{\circ} 20' 00''$

Moon's apparent Z. D. 72 26 30—0,0207202

Sun's apparent Z. D. 62 10 0—0,0533957

9,8083309

224 56 30 | 9,8861782

112 28 15

19,7686250

40 01 45 9,8843125, sin. of $50^{\circ} 0' 35''$
 50 18 15 $\times 2$

Its double is the dif. of azimuth 100 1 10

Supt. 79 58 50

2. R : t, $62^{\circ} 12'$ —10,2779915
 :: cos. dif. az. 100 1 10— 9,2405053
 : t, of 4th arc 18 15 45— 9,5184968
 Add the gr. Z. D. 71 34 40 as by Rule. II.
 Sum is 5th arc 89 50 25
 3. cos. 4th arc 18 15 45— 0,0224452
 : cos. 5th arc 89 50 25— 7,4452421
 :: cos. less Z. D. 62 12 00— 9,6687461
 : cos. red. dist. 89 55 18— 7,1364334
 Dist. by Eph. 90 24 16 at 4h. 3 om.
 Dif. 28 58 *

* The hly. decr. of }
 dist. by the Eph. } $30' 26''$
 As $30' 26''$: 15° long.
 :: 28 58 : $14 16\frac{1}{2}'$ W. long.
 (as appears by remark 2, p. 11)
 which added to the long. the ship
 was assumed to be in ;
 Viz. to $77^{\circ} 52'\frac{1}{2}$ W.
 Correction add 14 16 $\frac{1}{2}$ W.
 Ship's Long. 92 09 W.

Taken out at sight from
 Gardiner's Logarithms.

2. Method by Prop. 1. Sherwin's Tables of versed Sines, See p. 38 of 3d Edit. Having found their difference of azimuth, by the first Canon foregoing; or by Sherwin's Rule, p. 31, the angle at the zenith: And having given the two sides, and contained angle, of the spheric triangle; to find the third side.

Log. versed sine of the dif. of az. supt.	79° 58' 50" — 9,9169893	Rad. = 20000,000
Log. sine of Sun's tr. Z. D.	62 12 00 — 9,9467376	133°.46'.40" sum of sides
Log. sine of Moon's tr. Z. D.	71 34 40 — 9,9771534	46.13.20 supt. its
		na. v. fin. = 3081,368 and
		its supt. 16918,632

No. to Log. is 6932,350 — 9,8408803
 Nat. v. sine of 133° 46' 40" } — 16918,632
 the sum of the sides

Dif. is the nat. versed sine of - 9986,282 of 89° 55' 17" the red. dist.
 for 9985,456 — 89 55 00

If 2908 give 60" then will 826 give 17"

Dist. by Eph. at 3h. 0' — 91° 9' 55" } By Art. p. 154 of proportional Log. or in the
 Dist. by Obs. at 4 30 — 89 55 17 } book of Requisite Tables.

Dif. 1 14 38 prop. log. 3823
 Decr. of dist. in 3h. 1 31 17 its ditto 2949

Dif. 874 gives 2h. 27' 11¹/₂"

The time of the preceding dist. by Eph. add 3 0 0
 True time of the Observation at Greenwich 5 27 11¹/₂ P. M.
 True time of ditto at the ship before noon 0 30 00 A. M.

The time converted, } gives the Longitude 89° 18' W. = 5 57 11¹/₂"
 By table, p. 8.

I must recommend the Book of Logarithms, sold by Mr. Nourse in the Strand, which has the tables of sines and tangents to every ten seconds. And the first and last 4° of the quadrant, to each single second of the Sine, co-sine, tangent and co-tangent.—Quarto, bound in calf and lettered, price £2. 10s. Since the fourth Edition of Sherwin's Logarithms is worse than the third, and the fifth much worse than the fourth; this once valuable book is now of little use, owing to the bad management of the conductors.

Note, When the index of the log. of a sine, co-sine, or versed sine is 9, there will be four places of whole numbers in the natural number corresponding.

When the index is 8 there will be 3 whole numbers,

7	—	2 ditto
6	—	1 ditto
5	—	0 all decimals.

In Sherwin's Tables, where the radius of the log. sines is 10, the natural number to it, is 10000.

3. CALCULATION by Mr. DUNTHORN's Method. See Requisite Tables, p. 64 and 65, also the Appendix to the Nautical Eph. for the Year 1772, page 38.

Sun's apparent alt.	27° 50' 0"	Sun's true altitude	27° 48'
Moon's apparent alt.	17 33 30	Moon's true altitude	18 25 20
Difference	10 16 30	its nat. cof.	9839,64
Observed distance	90 20 —	its nat. cof.	58,18
Supt.	89 40 —		

Because the distance exceeds 90° the sum 9897,81 = 3,9955387

Log. difference by Table II. Subt. 10887

No. to Log. 9852,60 - - = 3,9935500

Nat. cof. of dif. of true alts. 9° 22' 40" 9866,35

This difference 137,5 is the nat. cof. of 89° 55' 16",
 the reduced dist. by the observation; as by the foregoing method.

4. CALCULATION by Mr. Lyon's Method. See Requisite Tables, p. 19 and 20, the Rules.

By Table 1. Large Sheet.

Agt. 27 deg. under 17 deg. is 0993 and under 18 deg. is 0843			
Agt. 28 under 17 is 1000, and under 18 is 0038			
1 ^o difference,		gives	61 and 96
If 60' : 61' :: 50' : 51' for the increase of greater alt. }			hence from 0939
If 60 : 96 :: 53½ : 54 for the decrease of lesser alt. }			take 3
Difference		3	No. to Log. 124''—2,0936
	deg. m. sec.		
Observed distance	90 20 00		Moon's hor. parallax 57' 20''—0,49680
Effect of refraction	+ 2 4		Log. cofec. of 27.48—10,33125
Dist. cleared of refrac.	90 22 04		Log. fine of 90.22—9,99999
Effect of parallax	— 26 52		Propl. log. 1st arc 26.45 — 0,82804
Reduced distance	89 55 12		Moon's hor. parallax 57' 20''—0,49680
Which is the same as by the other two methods, nearly.			Log. cofec. of 17.30.30—10,52166
		t. of 90.22. 0—12,19384	
			Propl. log. 2d arc 0. 0. 7 — 3,21230

5. Method—The Royal Astronomer's Rules and Method of Calculation, See the Appendix to the Nautical Ephemeris for the Year 1773, p. 2 to 18.

6. Method—Mr. G. Witchel's Rules and Method in said Appendix, p. 18 to 25.

THE Use of the following Table, is to reduce the difference between the reduced distance of the observation, and that by the Ephemeris, into longitude.

Enter the Table with the hourly increase or decrease of distance, or the nearest to it, at the head of the Table (the hourly difference is obtained from the Ephemeris), and in the column under it, find the minutes and seconds, or the nearest thereto, of the difference between the reduced distance and that by the Ephemeris at the time of observation, and in the right hand column you will have the longitude corresponding in the same horizontal line; which must be added to, or subtracted from the longitude you assumed the ship to be in, as directed in the first or second Remark, p. 11.

Ex. 1. Let the dif. be 1'.10'' as in the 1st Ex. p. 22, and first assumption; enter the Table under 28' and 28½', and you will find 1'.6'' and 1'.15'', the mean is 1'.10'' nearly, which will give 0°.37' in the right hand column under Lon.

2. The dif. 27'.5'', which dif. is not to be found in the Table, but it may be taken out at different times, I therefore take out for — — — — 13'.11'' the mean under 28' and 28½'.

Thus 13'.11'' gives 7°.00' lon.

For 13 .11 more 7 .00

For 0 .43 — 0 .24

For 27 .05 is 14 .24

From 27 .05

Take 13 .11

1. rem. 13 .54

2. rem. 0 .43

Ex. 3. The dif. 15'.18''

For 13 .11 is 7°.00' lon.

For the rem. 2 .07 is 1 .08

8 .08 which by the proportion in p. 23, is 8°.07'.

A TABLE

A TABLE to convert the Difference between the REDUCED DISTANCE, and that by the EPHEMERIS, into LONGITUDE of the EQUATOR.

The Hourly Increase or Decrease of Distance, being as in the Head Column.													Differ. of lon.
26'	27'	28'	28 $\frac{1}{2}$ '	29'	29 $\frac{1}{2}$ '	30'	30 $\frac{1}{2}$ '	31'	31 $\frac{1}{2}$ '	32'	32 $\frac{1}{2}$ '	33'	
0.08 $\frac{2}{3}$	0.09	0.09 $\frac{1}{2}$	0.09 $\frac{1}{2}$	0.09 $\frac{2}{3}$	0.10	0.10	0.10 $\frac{1}{3}$	0.10 $\frac{1}{3}$	0.10 $\frac{1}{2}$	0.10 $\frac{2}{3}$	0.11	0.11	5
0.17 $\frac{1}{3}$	0.18	0.18 $\frac{1}{3}$	0.19	0.19 $\frac{1}{3}$	0.20	0.20	0.20 $\frac{1}{3}$	0.20 $\frac{1}{3}$	0.21	0.21 $\frac{1}{3}$	0.21 $\frac{1}{2}$	0.22	10
0.26	0.27	0.28	0.28 $\frac{1}{2}$	0.29	0.29 $\frac{1}{2}$	0.30	0.30 $\frac{1}{2}$	0.31	0.31	0.32	0.32 $\frac{1}{2}$	0.33	15
0.35	0.36	0.37 $\frac{1}{2}$	0.38	0.38 $\frac{2}{3}$	0.39 $\frac{1}{2}$	0.40	0.40 $\frac{2}{3}$	0.41 $\frac{1}{3}$	0.42	0.43	0.43	0.44	20
0.43	0.45	0.46 $\frac{1}{2}$	0.47 $\frac{1}{2}$	0.48 $\frac{1}{3}$	0.49 $\frac{1}{4}$	0.50	0.50 $\frac{1}{2}$	0.51 $\frac{1}{2}$	0.52 $\frac{1}{2}$	0.53 $\frac{1}{2}$	0.54	0.55	25
0.52	0.54	0.56	0.57	0.58	0.59	1.00	1.01	1.02	1.03	1.04	1.05	1.06	30
1.01	1.03	1.05 $\frac{1}{2}$	1.06	1.08	1.09	1.10	1.11	1.12	1.13 $\frac{3}{4}$	1.15	1.16	1.17	35
1.10	1.12	1.14 $\frac{2}{3}$	1.15 $\frac{3}{4}$	1.17	1.18 $\frac{1}{2}$	1.20	1.21 $\frac{1}{2}$	1.23	1.24 $\frac{1}{2}$	1.26	1.27	1.28	40
1.18	1.21	1.24	1.25	1.27	1.28 $\frac{1}{2}$	1.30	1.31 $\frac{1}{2}$	1.33	1.34 $\frac{1}{2}$	1.36	1.37 $\frac{1}{2}$	1.39	45
1.27	1.30	1.33 $\frac{1}{3}$	1.34	1.36	1.38	1.40	1.41	1.43	1.45	1.46	1.48	1.50	50
1.35	1.39	1.42 $\frac{2}{3}$	1.44	1.46	1.48	1.50	1.52	1.54	1.55	1.57	1.59	2.01	55
1.44	1.48	1.52	1.54	1.56	1.58	2.00	2.02	2.04	2.06	2.08	2.10	2.12	
2.01	2.06	2.11	2.13	2.15	2.17	2.20	2.22	2.25	2.27	2.29	2.32	2.34	10
2.19	2.24	2.29	2.32	2.35	2.37	2.40	2.43	2.46	2.48	2.50	2.52	2.56	20
2.36	2.42	2.48	2.51	2.54	2.57	3.00	3.03	3.07	3.09	3.11	3.14	3.18	30
2.53	3.00	3.07	3.10	3.13	3.16 $\frac{1}{2}$	3.20	3.24	3.27	3.30	3.33	3.36	3.40	40
3.11	3.18	3.25	3.29	3.33	3.36 $\frac{1}{2}$	3.40	3.44	3.48	3.52	3.54	3.58	3.62	50
3.28	3.36	3.44	3.48	3.52	3.56	4.00	4.04	4.08	4.12	4.16	4.20	4.24	
3.45	3.54	4.03	4.07	4.11	4.16	4.20	4.24	4.29	4.33	4.38	4.41 $\frac{1}{2}$	4.46	10
4.03	4.12	4.21	4.26	4.30	4.36	4.40	4.45	4.50	4.54	4.59	5.03	5.08	20
4.20	4.30	4.40	4.45	4.50	4.55	5.00	5.05	5.10	5.15	5.20	5.25	5.30	30
4.37	4.48	4.59	5.04	5.09	5.15	5.20	5.25	5.31	5.36	5.41	5.46	5.52	40
4.55	5.06	5.17	5.23	5.29	5.34	5.40	5.46	5.51	5.57	6.02	6.08	6.14	50
5.12	5.24	5.36	5.42	5.48	5.54	6.00	6.06	6.12	6.18	6.24	6.30	6.36	
5.30	5.42	5.55	6.01	6.07	6.14	6.20	6.26	6.34	6.39	6.46	6.51 $\frac{1}{2}$	6.58	10
5.47	6.00	6.13	6.20	6.27	6.34	6.40	6.47	6.54	7.00	7.07	7.13	7.20	20
6.04	6.18	6.32	6.39	6.46	6.53	7.00	7.07	7.14	7.21	7.29	7.35	7.42	30
6.21	6.36	6.51	6.58	7.15	7.13	7.20	7.27	7.34	7.42	7.49	7.56	8.04	40
6.39	6.54	7.09	7.17	7.35	7.33	7.40	7.47	7.55	8.03	8.10	8.18	8.26	50
6.56	7.12	7.28	7.36	7.44	7.52	8.00	8.08	8.16	8.24	8.32	8.40	8.48	
7.14	7.30	7.47	7.55	8.03	8.12	8.20	8.28	8.36	8.45	8.54	9.02	9.10	10
7.32	7.48	8.05	8.14	8.23	8.32	8.40	8.49	8.57	9.06	9.15	9.23	9.32	20
7.48	8.06	8.24	8.33	8.42	8.51	9.00	9.09	9.18	9.27	9.36	9.45	9.54	30
8.06	8.24	8.42	8.52	9.01	9.11	9.20	9.29	9.38	9.48	9.58	10.06	10.16	40
8.23	8.42	9.01	9.11	9.21	9.31	9.40	9.49	9.59	10.09	10.19	10.28	10.38	50
8.40	9.00	9.20	9.30	9.40	9.50	10.00	10.10	10.20	10.30	10.40	10.50	11.00	
9.32	9.54	10.16	10.27	10.38	10.49	11.00	11.11	11.22	11.33	11.44	11.55	12.06	30
10.24	10.48	11.12	11.24	11.36	11.48	12.00	12.12	12.26	12.36	12.48	13.00	13.12	
11.16	11.42	12.08	12.21	12.34	12.47	13.00	13.13	13.26	13.39	13.52	14.05	14.18	30
12.08	12.36	13.04	13.18	13.32	13.46	14.00	14.14	14.28	14.42	14.58	15.10	15.24	
13.00	13.30	14.00	14.15	14.30	14.45	15.00	15.15	15.30	15.45	16.00	16.15	16.30	30
13.52	14.24	14.56	15.12	15.28	15.44	16.00	16.00	16.32	16.48	17.04	17.20	17.36	
14.44	15.18	15.52	16.09	16.26	16.43	17.00	17.17	17.34	17.51	18.08	18.25	18.42	30
15.36	16.12	16.48	17.06	17.24	17.42	18.00	18.18	18.36	18.54	19.12	19.30	19.48	00
16.28	17.06	17.44	18.03	18.22	18.41	19.00	19.19	19.38	19.57	20.16	20.35	20.54	30

If you cannot take for the dif. of the distances out at once, you may take it out of the same column at 2 or 3 times, and the sum will be the longitude resulting.

Of

Of the SOLAR SYSTEM.

THE parallax* of the sun deduced from the observations made the 6th of June, 1761, on the day of the transit was $8''.565$; and on the 3d of June, 1769, on the day of the transit, it was $8''.65$, which reduced to the mean distance of the sun from the earth, gives the mean parallax of the sun $8''.75$ or $8''.78$. The late ingenious and excellent Astronomer, Mr. James Short, deduced the sun's parallax from a great number of calculations; the mean of 53 calculations gave $8''.61$; the mean of 63 gave $8''.63$; the mean of 21 gave $8''.56$ and $8''.57$ on the supposition that the sun's parallax was $8\frac{1}{2}''$.

And the Reverend Mr. Hornsby, Professor of Astronomy at the University of Oxford, by supposing the sun's parallax $9''$ deduced from a mean of 14 calculations $8''.69$ and $8''.65$; whose mean is $8''.67$ from the observations of the year 1761. And from the observations of 1769 (on the day of the transit) the mean result, gave the parallax $8''.65$; from which this Reverend Professor, by the foregoing results, gets $8''.78$ the sun's parallax, at a mean distance from the earth; and the dimensions of the solar system.

The DISTANCES of the PLANETS from the SUN, as under.

The Planets	Relative distances	Absolute Distances
Mercury	387,10	36,281,700
Venus	723,33	67,795,500
Earth	1000,00	93,726,900
Mars	1523,69	142,818,000
Jupiter	5200,98	487,472,000
Saturn	9540,07	894,162,000

By means of an acromatic object glass micrometer 40 feet focus, which Mr. J. Short adapted to a reflecting telescope of two feet focal length, he measured the greatest and least diameters of the sun, and found the apogee diam. = $31' 28''$ and the perigee diameter = $32 33$

The mean of them	32 00,5
It's half is	16 00, $\frac{1}{4}$

We suppose the fixed star Sirius to be 27000 times further from us than the Sun.

To find the DIAMETER of the SUN in English Miles.

In Fig. 6. Let E, represent the centre of the earth; S, that of the sun; and UL, his diameter. Then in the right angled triangle, ESU, are given ES, and the angle UES, to find SU.

As R. : dist. $93726900 = 7,9718643$
 :: s, of $\frac{1}{2}$ diam. $0^\circ.16'.00'',25 = 7,6679575$

: $\frac{1}{2}$ the diam. $436336,5 = 5,6398218$

Hence 872673 is the diam. in

— English miles.

And the equatoreal diameter of the earth being 7970 miles; as deduced from the measures of a degree of the arches of meridians. Hence $7970 | 872673 | 109\frac{1}{2}$ times, that the diameter of the sun is greater than the earth's. But from a mean of a number of the measured

diameters of the sun, with micrometers, there results for the diameter. $32'02''$, and of the planets, reduced to the mean distance of the earth from the sun, as are in the following table; and from which I have calculated their absolute diameters, circumferences, and surfaces. The mean angular diameter of the sun $32'.02''$, gives his diameter 109,58 times that of the earth's.

On

* The parallax of the sun, moon, or star, is the greatest at the equator, that semidiameter being the greatest.

On the day of the transit 1769, the relative distance of the Sun from the earth was - - - - - 101512,6
 And of Venus from the sun 72626,3
 Of Venus from the earth 28886,3
 Whence, if 101512,6 give 8'',6; the relative mean distance 100000 will give : 8'',73
 and if ditto give 8,65; ditto 100000 will give 8,78
 the mean is 8,755;

but we will take $8''\frac{1}{4}$ for the parallax of the sun at the mean distance from the earth:

And it will be, as 28886,3 : 58'' :: 100000 : 16'',75388

and ——— : 57,8 :: ——— : 16,69628

mean 16,725 diameter of Venus.

Log. of sun's mean dist. 7,9718643

s, of $\frac{16,74}{2} = 8'',36 = 5,6070798$

$\frac{1}{2}$ diam. of Venus 3792,66 = 3,5789441

diam. of Venus 7585,32

Log. of 3,1415926536 = 0,4971499

Diam. 7585,3 = 3,8799728

Periphery 23830 = 4,3771227

Surface 180757125 = 8,2570955

By the same method of calculation I have obtained the diameters, peripherys and surfaces of the other planets.

Diameters reduced to the mean dist. of the sun from the earth,	Equatoreal diameters in minutes.	Peripherys in miles.	Surfaces square miles.
Of the sun - 32' 02" —	873355,4	2743722 $\frac{1}{2}$	2,396,241,111,110
— earth - ———	7970,0	25038 $\frac{1}{2}$	199,556,845
— moon - 0' 04'',915	2226,0	6993,2	15,566,836
— Mercury 0 7	3148,2	9890,4	31,136,843
— Venus - 0 16,7	7585,3	23830,0	180,757,125
— Mars - 0 11,4	5162,5	16208,5	83,728,000
— Jupiter - 3 13,7	88000,0	276460'0	24,895,186,000
— Saturn - 2 51,7	78020,0	245107,0	19,123,253,000
— Saturn's ring 6 40,6	181980,0	571707,0	104,040,000,000

And 109,58 cubed is 1315812 the bulk of the sun, to the earth 1*.

RATIO of the PLANETS, to that of the EARTH being 1.

Diameters.	Surfaces, that of the earth's.	
☉ 109,58	11982 times greater than the earth's	☉ the Sun
⊖ 1,00	— 1	⊖ the Earth
☾ 0,28	— $0\frac{1}{3}$ or 13 times less than the earth's	☾ the Moon
☿ 0,395	— $0\frac{1}{6}$ or 6 times less than the earth's	☿ Mercury
♀ 0,952	— $\frac{1}{10}$ or $\frac{1}{10}$ less than the earth's	♀ Venus
♂ 0,648	— $\frac{4}{10}$ or $\frac{4}{10}$, not half the earth's	♂ Mars
♃ 11,04	— 125 times the earth's surface	♃ Jupiter
♄ 9,79	— 96 times ditto	♄ Saturn.
Ring of ♄ 22,836	— 520 times ditto	

G

RE-

The Sun is 1 million, 315 thousand, 812 times bigger than the Earth.

REMARKS. Of the DECREASE of the OBLIQUITY of the ECLIPTIC.

1. The annual precession of the equinoxes being $50''.3$ as by Mr. Mayer, or $50'.18''$ in 60 years. The obliquity of the ecliptic in the year 1756, was $23^\circ.28'.16''$; and in the year 1790, it will be $23^\circ.28'$. And, since the decrease of the obliquity is $1'$ in $130\frac{3}{7}$ years, or $7'$ in 913 years, therefore in 7826 years, the obliquity will be 1° less; and in 180000 years, the obliquity will be 23° less; and therefore only $28'$ obliquity, or the greatest declination that the sun will then have, according to the tables of our modern astronomers.

2. When this happens, the days and nights will be equal all over the habitable earth, all the year round, and for many thousand years, the variation will be very little, since the decrease is so little as 1° in 7826 years. Whence in the year 181790, the obliquity will be only 28 minutes. And in the year A. C. 185443, the planes of the equinoctial and ecliptic will coincide, and therefore no obliquity, nor variation in the length of days.

FIG. 7, is a representation of the planetary system.

S, the sun in the centre of the universe, i. e. of this System.

	Period round the sun, relative to the 1st point of Aries.				
M ☿ a, the Orb of Mercury	87	days	23 h. 14'	20"	As $365^\circ\frac{1}{4} : 50''.3 :: 88$ days : $12''.1$ that the stars advance in the time that ☿ makes a revolution, which gives 1 m. 10 sec. of time, to be sub- tracted from its siderial period, to obtain the true time of its
L ♀ b, the Orb of Venus	224		16 41	37	
K ⊕ E, — of the Earth	365		5 48	48	
I ♂ D, — of Mars	686		22 19	00	
P ♃ — of Jupiter	4330		12 27	00	
Q ♄ — of Saturn	10747		2 50	00	
r s t, the sphere of the fixed *'s					

period to a fixed point. See De la Caille's Astronomy, sect. 3, art. 6. These are called tropical periods. The time of the moon's periodic revolutions are longer when the earth is in its perihelion, than when in its aphelion. The mean periodic revolution of the moon about the earth is 27d. 7h. 43m. and the synodic 29d. 12h. 44m.

1. That ☿ and ♀, revolve about the sun, is evident from their moon like appearance. When they shine with a full face, they are in respect of us beyond the sun; when they appear half full, they are at about the same distance as the sun; when horned they are nearer to us than the sun; and when either of them passes between the sun and us, and has then only a small quantity of lat. the planet will then appear like a spot on the sun, as it transits over his disk.

2. That ♃, ♃, and ♂ surround the sun and the earth, is evident by their shewing full faces when near a conjunction with the sun; and the shadows of the satellites of ♃ and Saturn that appear sometimes upon their disks, make it plain that the light they shine with, is not their own, but borrowed from the sun. See Motte's translation of Newton's Principia, vol. 2, p. 209, and the following pages. The primary planets, by radii drawn to the earth, describe areas no wise proportional to the times, but that the areas which they describe by radii drawn to the sun, are proportional to the times of description. The moon by a radius drawn to the earth's centre, describes areas proportional to the times of description nearly.

P. 220, Prop. 6. Theo. 6. That all bodies gravitate towards every planet, and that the weights of bodies towards the same planet, at equal distances from the centre of the planet, are proportional to the quantities of matter which they severally contain.

Prop. 21. The equinoctial points go backwards, and the axis of the earth, by a nutation in every annual revolution twice vibrates towards the ecliptic, and returns to its former position.

Prop. 24. The flux and reflux of the sea, arise from the actions of the sun and moon. The force of the moon to that of the sun, in raising the tides, are as $4\frac{1}{2}$ to 1 nearly.

Prop. 39. To find the precession of the equinoxes.

To find the ALTITUDE of an OBJECT, as a House, a Tree, an Obelisk, or the like: Also to find the ALTITUDE of the Sun or Moon, by observing with a HADLEY's Quadrant, Sextant or Octant, and an Artificial Horizon.

ARTIFICIAL HORIZONS may be of various Kinds.

1. **T**HE true plane glass floating on quicksilver: The box to hold about one pound of quicksilver, for the glass to float upon, and it may be either round, square, or oblong.

2. The glass plane, mounted in wood or brass, with three screws to raise or depress it so as to set it level by means of a spirit level.

3. A basin of water will do very well where the observation can be made out of the wind, and so that its surface be at rest, or without motion.

The principles of this method of observation may be demonstrated by the following figure.

The ARTIFICIAL HORIZON adapted for the taking ALTITUDES at Sea, is as follows.

A piece of mahogany wood, one inch thick and 10 inches square, with an oblong part cut out from its middle, about 4 by 3 inches, and $\frac{6}{10}$ of an inch deep; with a hole made in one corner, for to let the quicksilver out of that basin, into a bottle, through a hole passing through one of the feet, which is so made as to go a little way into the mouth of the bottle; and when it is to be used the peg is to be put into the hole; and then pour the quicksilver into the basin, and put the glass plane upon it; about this basin is a groove, the depth of the basin, and about $\frac{3}{4}$ of an inch wide, for receiving the quicksilver when it happens to be thrown out of its basin by the jerks of the sea. Some people use it in this manner; but if they were to have a frame made, such as that its parallel ends (whose base is about 8 inches) and the sides of these ends of the frame, to make a right angle at the upper end of the frame where the parallelogramic sides meet; in these sides are to be fixed two glasses of a convenient size, for the rays of the sun to pass through, and to fall fully on that which lies upon the quicksilver; and so, that in the time of observation you may observe the sun's image reflected from the index glass, to the object glass, and from thence to the eye, to be made to appear to in the same line as the image of rays which come from the sun, and passes through the glass of the frame in the further side, and falls upon the floating plane, and from it, is reflected to the eye; and when these images are made to appear to be coincident, or face upon face, then you will have the double altitude cut with the index of your reflecting instrument.

Ex. June 13th 1778, at Dock Town, Plymouth: Its Lat. $50^{\circ} 22' \frac{1}{2}$ N. which is $1' N.$ from Drake Isle.

Double alt. obs. at $45^{\circ} 30'$	Z. D. $67^{\circ} 17'$	Tr. az. $77^{\circ} 38' NW.$
App. alt. $22 \ 45$	C. L. $39 \ 39$	Mag. $55 \ 40 NW.$
Ref. 2	P. Dist. $66 \ 40$	Variation $21 \ 58 W.$
True alt. $22 \ 43$	The 14th in the Morning Var. $22 \ 20 W.$	
	The Sum $44 \ 18$	
	The half sum is the true variation $22 \ 09 W.$	

In Plymouth Sound I found the var. by taking the mean of 12 observations, to be $22^{\circ}.08' W.$

In the year 1772, the variation, as observed by Messrs. Wales and Bayly, was $21^{\circ}.13 W.$ high water in the found - 5 h. 15 m. full and change day.

And at the dock 5 45

The

Note. The glasses in the frame to be true ground, and their planes parallel.

OF SURVEYING COASTS and HARBOURS.

THERE are various methods, and various kinds of Instruments, with which this is performed.

Thus, Land Surveyors make these surveys by means of using a Gunter's chain, and theodolite, or plane table; but seamen generally use the azimuth compass for taking bearings; and a log-line, in length 50 or 60 fathoms, or more, together with an offset staff, for taking distances, from places in the right line they are measuring, between one station and another, to the coast or high water mark. These dimensions are kept in columns as in the following Field-Book. Having viewed the coast to be surveyed, and remarked the places for the stations, also provided a number of object rods or staves, which may be of fir deal, six feet long, and $1\frac{1}{2}$ inch square, with a number of boards about a foot square, painted white, and with a wooden socket to each, so that they may be made fast upon the tops of such object staves, as occasion may require, and when a staff is fixed at a place, let it be so that the board may face the station from whence it is to be viewed; dispose of the other rods in like manner.—Some tie pieces of white bunting to the top of the rods, instead of boards.

In this measurement, there should be 3 or 4 assistants, to wit, the principal with his azimuth compass (with a card that traverses well, and that has a nonius or stop to its card), one assistant to lead the line, and another to follow it, who is to take up the pricks, which the first puts down at the end of each line; the third assistant to measure perpendicularly from the line to such points of the high and low-water marks of the coast as are necessary, the fourth to take up the rods when done with at any place, and so on from rod to rod, on the coast; another assistant with his rods, and men to attend him in a boat, at low water time, to erect them upon the extremes of sands and rocks, and to be removed from place to place, according to signals given him from on shore.

A marine mile contains 6072 feet = 2024 yards = 1012 fathoms English.

But 1760 yards : 2024 yds. :: 72 inches : 82,8 the inches in a fathom, sea measure; and 880 such fathoms in 1 sexagenary mile, and 60 to 1°: hence 1 fathom contains 6 feet 10 inches and $\frac{8}{10}$. And because all sea charts are plotted by a decimal scale of miles, it will therefore be necessary for the measuring line (properly adapted) to be divided into decimal parts of a mile. I shall therefore recommend the length of the line to be used, to be the hundredth part of a sea mile, which } Therefore the decimal fathom is 72,864 inches.

is 60,72 feet = 1 line	Also the decimal parts of a line	
607,2 = 10 lines	$\frac{1}{10}$ or, 1 of a line = 6,072 feet	,6 of a line = 36,43
6072 = 1m. = 100 lines	,2 = 12,14	,7 = 42,50
	,3 = 18,21	,8 = 48,57
	,4 = 24,28	,9 = 54,64
	,5 = 30,36	

Some use a log-line, each knot = 50,6 feet.

12 knots = $\frac{1}{10}$ of a mile.

120 knots = 1 mile.

But the line which is $\frac{1}{100}$ th part of a sea mile, or 60 feet, 8 inches, and 6 tenths long, being divided into 10 equal parts, is a decimal line of measures.

The off-set staff = 6 ft. 0,86 in. or 72,86 inches.

H

Suppose

Suppose you measure with a line of 2, 3, or 4 times the length of 60,72 feet, it will be best to reckon each part of such line to be a line, this line may be marked with a piece of packthread at each fathom, by tying as many knots on the thread as it is fathoms from the aft end of the line, and at the tenth a piece of strong leather with one string tied to it, and at the twentieth to tie two strings, and so on to the other end; then your line will be decimally divided.

Thus 10 parts is 1 line	=	.01 of a mile
10 lines is $\frac{1}{10}$ of a mile	=	.1
And 100 lines is	=	1.0 mile.
	miles	m. lin. pts.
Hence 4,77 exprefs	4	7 7
And 0,64 exprefs	0	6 4

By this method the sea coast measure will agree with the sea chart measure; but the land-surveyor's measure of coasts in miles of 1760 yards each, is the cause why by sailing near a coast from one point of land to another that the sea and land measures so much differ.

Remark that the base for obtaining distances should not be less than one tenth or one twelfth of the dist. of the station where you are, to the place of which the dist. is required; also when the bearing of any object makes a small acute angle with the base, from both extremes of it, that the place of such object will be badly determined, by reason of the intersection of the two position lines being so acute at their meeting.

Sea coasts in most places are composed of rocks, hills and valleys, and in such circumstances base lines cannot be measured along the coast from station to station, otherwise than measuring as the ground will admit (to by stations), as will be shewn in the following survey and plan: But after all that can be said, much will depend upon the judgment of the artist, as a great many varieties will happen, in making these surveys, that cannot be described within such small limits as are here presented; yet if what is here said, be well understood, the artist may seldom want any thing further on this head.

A FIELD-BOOK of the survey of the Coast from Chicester harbour mouth to the Bill of Selfey, and from thence across Selfey harbour to Arundel harbour; together with the position lines, or bearings of various parts of sands and rocks to the Southward of the Bill called the Male Owers, which is a hard black rock nearest to the Bill; and South of it three quarters of a mile, is Piller Rock; and EbN. of Piller Rock, are the Shoals Rocks about 2 miles and a half; between the Malt and Piller Rocks runs a stream called the Looe Stream; and SSE, from Shoals Rocks is East Borrowhead and the Sea Owers a black rock; over this rock runs a prodigious current, the flood tide sets over it ENE.

At the first station, with my azimuth compass fixed upon its three-legged stand, five feet high, and with the North point of the card 22° NW. (upon the top of this stand is a square part, with a rim which stands up about an inch to keep the wooden box of the compass firm).

F I E L D-

F I E L D - B O O K.

The first station line bears - - -	42°.40' SE.	Dist. 50 lines Offsets to H. and L. water marks. Cockbush house. 23,4 lines and parts to coast. 48,7 low water mark. 2,1 H.W. bank 1,5 do. 21,5 do. + 20,5 to L.W. do. is 42.	10 parts make 1 line 100 lines = 1 mile.
An isle - - - - -	43°.03' SW.		
The isle is 2 lines long and 1 broad	Distances 40,1 50,4		
Note, the true meridian is 22°.30' E. from the magnetic North.	136 276 300		
2d station line bears - - - - -	At $\odot$ 2. 24°.30' SE. 30 138 162,5	The isle bears 52° NW. to 3d station. 20 to H.W. bank. 12 do. 21 do.	
3d station line bears - - - - -	$\odot$ 3 3°.10' SE. 45 57 120	53 to the tail of Selfey harbour. 20 to H.W. bank. 2 to do. 6,6 to do.	
To return to this - - - - - 4th station line bears - - - - -	$\odot$ 4. 37°.50' SE. 38 62	The church <i>b</i> and mill <i>b</i> , in one. 11 to H.W. bank. 13 to do.	
5th station line bears - - - - -	$\odot$ 5. 82°.00' NW. 50,3 60,7	5. 26. 16.	
6th station line bears - - - - -	$\odot$ 6. 67°.10' NE. 30,0 48,9 142,1	5,6 to bank. 19,7 to do. Fishermen's houses. 0 on the bank.	
To return to this station - - - - - 7th station line bears - - - - -	$\odot$ 7 55°.5' NE. 66	N. B. The bearings of the rock, &c. are to be taken from stat. 4 and 7. 16 to bank, which runs from last station, bowing to seaward.	
8th station line bears - - - - -	$\odot$ 8. 34°.10' NE. 90 120	17 15 to bank.	

 $\odot$ denotes station.

9th

9th station line bears - - - -	⊙ 9. E.	
Joins the neck of the harbour	3,3	
From station across the neck	18,0	6 to bank.
	40	4 to do.
	57	16
	80	10
	120	12
	140	21
	168	} The bank runs waving inwards and outwards, and these offsets are the least distances.
	180	
10th station line bears - - - -	⊙ 10. 87°.00 SE.	At 5 we passed the neck of Selfey harbour, which is 12 lines wide.
9 lines to Middleton church	160	21 passed 3 fishermen's houses..
And 4 to a large house	246	12 to bank } bending in and out
	363	10 to do.
11th station line bears - - - -	⊙ 11. 44°.00 SE.	
	50	12 to bank.
	60	1.
12th station line bears - - - -	⊙ 12. 77°.00 SE.	
	40	7 to bank.
	60	20.
	100	10.
13th station line bears - - - -	⊙ 13. 80°.30' SE.	
	46	4 to bank.
Little Hampton church is on the	123	11 to do.
East side of the haven.	127	Joins the haven of Arundel.

Returned to station 4. Bearings taken there.

The S. part of Chichester shoals 42° NW. in this line at 100 crossed the shoal; at 150 the shoal to right hand 42 lines, and at 260 to shoal 47 lines. At 386 joined the shoal. At 460 the line, left the shoal; and at 550 we were in direction of the E. part of the isle; this line was measured with the line and half minute glass, in the boat; after the soundings were taken upon the shoal, and poles put up at various places near the skirt of the shoal.—This method being well known to seamen, therefore no occasion to say any thing more about it. In like manner, having set up poles at such parts on the sands and rocks, whose bearing are to be taken, as follows: Pillar Rock, W. side bears 35° SW. The neck of this rock bears 7° SW. which is right across the Male Owers, the dist. between these is 1 mile and quarter; the Pillar Rock and Sand is 110 lines N. and S. and this extends to the Sea Owers and East Borrowhead, which is 5 miles and a half; as per plan.

Sea Owers, 22°.30' SE. which passes the West end of the innermost shoal rocks.

N. end of Borrowhead 37°.10' SE.	At ⊙ 7.	Returned to station 7.
The West part of Pillar Rock -	58°.00' SW.	Which line crosses the Male Owers
Neck of pillar	36.15 SW.	} The middle of the inner rocks and the Sea Owers, S. by compass.
W. end of Inner rocks and Sea Owers	10.35 SW.	
East part of the inner rocks -	10.10 SE.	
North end of East Borrow-head	15.25 SE.	
Seaward limit of Bogner rocks -	82.30 SE.	
Station 4 bears - - - -	88.00 SW.	Dist. 220 lines, or 2 miles and $\frac{1}{3}$.

OF COAST AND HARBOUR SURVEYING.

HAVING taken a view of the Place, and parts of the shore, the plan of which is to be drawn; also noted the places where your stations are to be, from which you are to take the bearings, and to measure the distances between the stations, and the nearer to a level the better; for if there be any ascents and descents, some allowance must be made in the measure: Remark, that you must see the place of the second station from the first, the 3d from the 2d, and so on—Also, that from some two of the stations, each signal and flag-staff erected on the sands, shoals and rocks, must be seen, in order that their bearings may be taken with the azimuth compass, and that the construction of the plan may be drawn from the intersections of the position lines; as will evidently appear by the following construction of the plan from the Field-book thereof.

Note, that some of the places on shore, in this plan, are denoted by small letters; those in and about the harbour and bay, with capital letters; and the places of the stations by figures, in order of the stations.

The first station line AB, is the principal base for obtaining the places of the sands, by the intersections of the position lines:—See the plan. Having fixed the azimuth compass upon a stool or stand, and finding its needle to traverse well, I then turned the compass till the thread (or the like) in the vane next the object, appeared to cut, or to be in the line of the direction of the Tower Church at *a*, which bore $44^{\circ}.30'$ NE.

The wind mill *b*— $88^{\circ}.40'$ NE. The extreme of the base B - - - $89^{\circ}.30'$ SE.

The signal on the sand *z*— 72° SE. flagstaffs on the sand *y* - - - 65.30 SE.

The signal on the North point of the long sand *x* - - - 61.20 SE.

The points T, of the sand of the island S - - - 61.20 SE.

The East and West parts of the island S - - - 55.00 SE.

The point C, at the West end of the sand - - - $47^{\circ}.00$ SE.

Measure of the first station line from A, towards B, with the offsets measured perpendicular to the base line, to the high and low water marks.

At—A to H.W. 2 to L.W. 10 fathoms.

Note. H.W. denotes high water mark
L.W. low water do.

At 50 — 1 — 16

At 125 — 5 — 10

R 306 — 20 — 25 at R, the Tower Church (*a*) bears N. by compass.

The N. point T, of the sand, and the staff U, in one, bears $29^{\circ}.30'$ SE.

The point C, of the sand, South.

					Bearings from B.—Point C of the sand 59°.20' SW.				
At	554	—	25	—	55	NW. point of the island	S	—	54.45 SW.
	762	—	21	—	51	The staff	V	—	47.05 SW.
	875	—	13	—	52	The point	W	—	31.20 SW.
	925	—	30	—	65	The point	X	—	17.00 SW.
	975	—	25	—	—	The W. point	Y	—	20.25 SE.
	1000	—	32	—	82	The point	Z	—	52.10 SE.
B	1070	—	25	—	110	At C, the staff	F	—	66.00 SE.
	1100	—	27	—	—	the staff	E	—	17.35 NE.
	1127	—	13	—	118	the staff	D	—	20.25 NW.
	1150	—	30	—	83				
C	1200	—	20	and joins C, the SW. point of the harbour.					

I

Measures

Measures from C to D, bearing $20^{\circ}.25'$ NW.

At 50 — 14 H.W. bank; 27 L.W.

125 — 12 — 24

175 — 20 — 27

D 260 — 15 — 32

D to $\odot$ 5, bearing 40° NW.

At 56 — 50 H.W.

$\odot$ 5 to 6, bearing $3^{\circ}.30'$ NE.

At 85 — 1 to H.W. bank.

126 — 3

136 — 25 joins H.W. bank, on the right, continued to 62, where it intersects the bank perpendicularly, the point runs to the right 25 fathoms.

175 — 76 to bank.

247 — 24

364 — 51

6 $\odot$ 450 — 115 to river, across the river 44, continued ESE. 52 to stat. 7.

$\odot$ 7 to 8, bearing $8^{\circ}.10'$ SE.

Bearing to the point of the peninsula 25° SW.

Measure 76—then to the right hand offset to River 26.

left hand do. to River 22 to the turn of the inlet.

136 to the end of the peninsula.

$\odot$ 7 to 8, At 56, to the turn of the inlet 27 to the right hand.

At 100, to river 25

185, to R. 35

235, to R. 25

285 to R. 36

290 to the left 2 to the SW. corner of the Town-warehouse on the Keys, Custom-house and town.

8 stat. — 235 to R. — 12
— 410 to R. — 9

Stat. 8 to 9 — 68 to R. — 4

$\odot$ 9 to 10 at 274 to R. — 27

303 to R. — 30

400 to R. — 18

Castle 424 to R. — 20

H.W. bank 484 to R. — 22

10 stat. — 526.

Bearings of, by stations, or points.

In 7 stat. line, at 86, the point of land on the W. side and stat. 5, in one.

At stat. 9, stat. D bears $68^{\circ}.30'$ SW.

Ditto church O - - 89.30 NE.

At F, the extremes of the sand y, is S. 11° W.
and S. $23\frac{1}{2}^{\circ}$ W.

The Eastern part of Long Island due S.

At G, the extremes of sand z, 15 and 24° SW.

At stat. 12, the W. and S. parts of Basin Isle 60° SE.

At stat. 14, the W. part of the island S. 3° W.

At 112 (b) is 14 stat. line, the W. end of I. S. 9° W.

At stat. 15, the S. part of the isle S. 52° W.

Stat. 10

Note, R. denotes river.

⊙ 10 to 11 at 100 to left 30 H.W bank to right 25 L.W. mark.

124 to do. 24

140 to do. 27

200 to do. 17—and to L.W. 34 on the right.

273 to do. 5

Sand z 350 to do. 2. The extremes of sand z bears

490 to do. 70 to the right, L.W. 50

586 to do. 56 do. 26

626 to do. 84 do. 20

11 flat. — 685 to do. 48 do. 14

Continued 736 to do. 36.

786 to do. 76 do. 16

830 to do. 37 joins the SW. point of the entrance into the winter harbour for the ships. Returned to flat. 11.

⊙ 11 to 12 at 86 to bank on the right 166

146 to do. 64 which bank runs in direction W. so as to meet

174 to do. 27 the last measure rounding,

247 to do. 21

12 flat. — 275

⊙ 12 to 13 at 23 to bank — 24

150 joins the bank.

⊙ 13 to 14 at 70 the mouth of the river, which runs into the bason.

⊙ 14 across the river 27 to the right, 36 to the bank.

At 136 to bason 34

At 200 to do. 14

15 flat. — at 248.

⊙ 15 to 16 - at 34 to bason 12

16 flat. — 90 do. 20

Stat. 16 to 17 at flat. to bason 26

at 90 to do. 16

17 flat. — at 128.

⊙ 17 to 18 at 30 to bason 20

18 flat. — at 85 } 17 station line continued

At 100 to do. 12—At 145 to bank of the bay 46, at 146 intersects the bank.

18 flat. to the Eastern part of the neck of the bason,

Offsets, left hand.

Distances from ⊙ 18.

Offsets, right hand.

To bank of bay — 10

— 30 —

46 to bank of the bason.

— 75 —

42

See the Sketch in } do. 53

— 110 —

38

the Field-book } 28

— 125 —

30

20 and 35

— 150 —

36

20

— 266 —

30 a rounding point at 276.

Dimen-

Dimensions taken upon Long Sand.

Offsets left side		At point X.		Offsets right hand side of station line.	
		Stat. 1, line 8° SE.			
To L.W. mark	25	—	25 —	76	to L.W. mark.
			47 —		
	50	—	80		
	52	—	150 —	63	
	76	—	200 —		
			270 —	20	
		Stat. 2 line.			
		55°.40' SE.			
To L.W. mark	86	at	90 —	27	
	100	—	416 —	27	
	106	—	526 —	28	
	77	—	600 —	34	
			616 —	Joins the E. end.	

Remark that 55°.40' SE.

Or S. 55.40 E.

denotes the same, and is so used by different authors.

Bearing of station lines.

From station	7 to 8	8°.00' SE.
	8 to 9	34.20 SE.
	9 to 10	21.15 SE.
	10 to 11	89.00 SE.
	11 to 12	— N.
	12 to 13	— E.
	13 to 14	55.00 NE.
	14 to 15	88.30 NE.
	15 to 16	37.00 SW.
	16 to 17	87.30 NE.
	17 to 18	20.00 SE.
From 18 stat. to the W. point of Peninsula.		74.00 SW.

1. Note. The figures wrote in the bay and in the harbour are the depths of water, at the time of low water, in fathoms.

2. The angular distances of objects, situated in respect of each other horizontally, may be taken with a Hadley's Sextant, or like instrument. For example:—From the point F, the $\angle yy = 12^\circ.30'$, and if the bearing of one of them be known, the other will also be known.

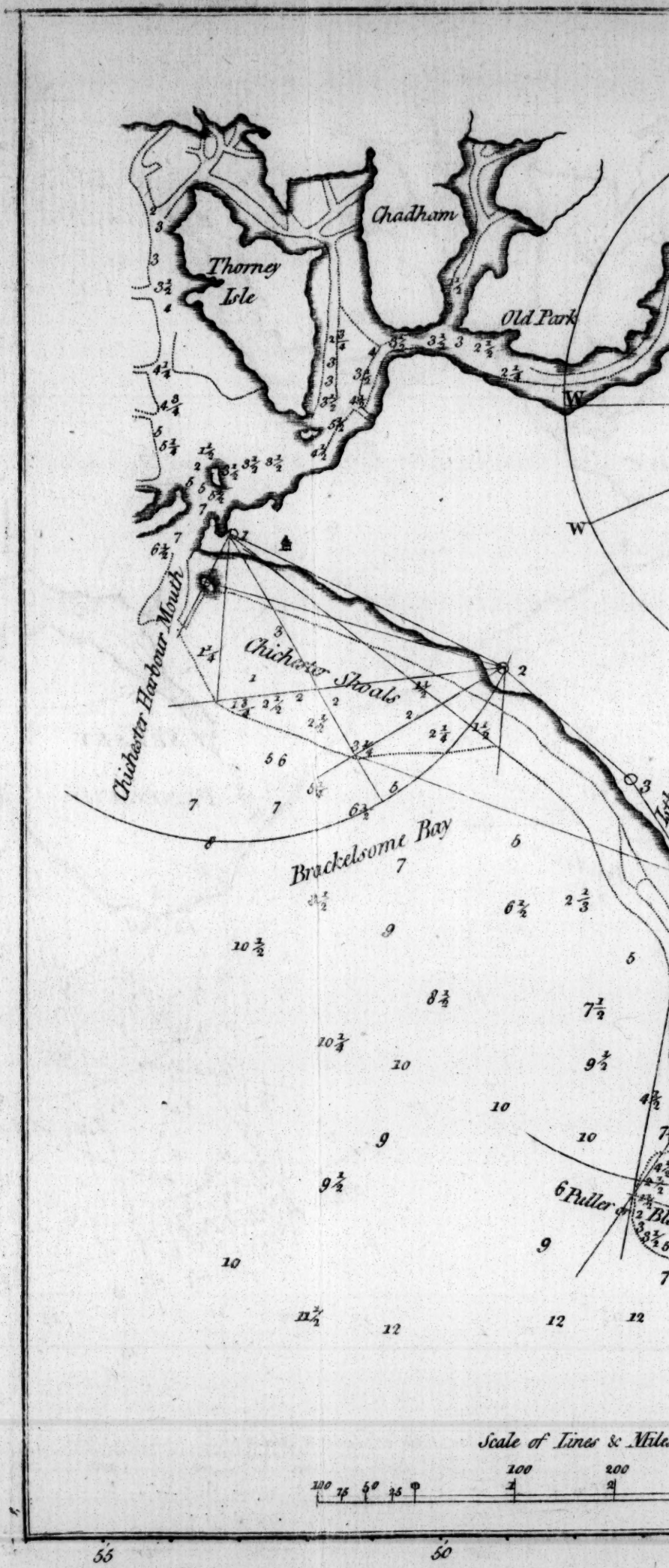
3. At R, the bearing of staff C, is S; the angles measured with a reflecting instrument at R, as CT, CA, CX, &c. will be their bearing in respect of the magnetic meridian.

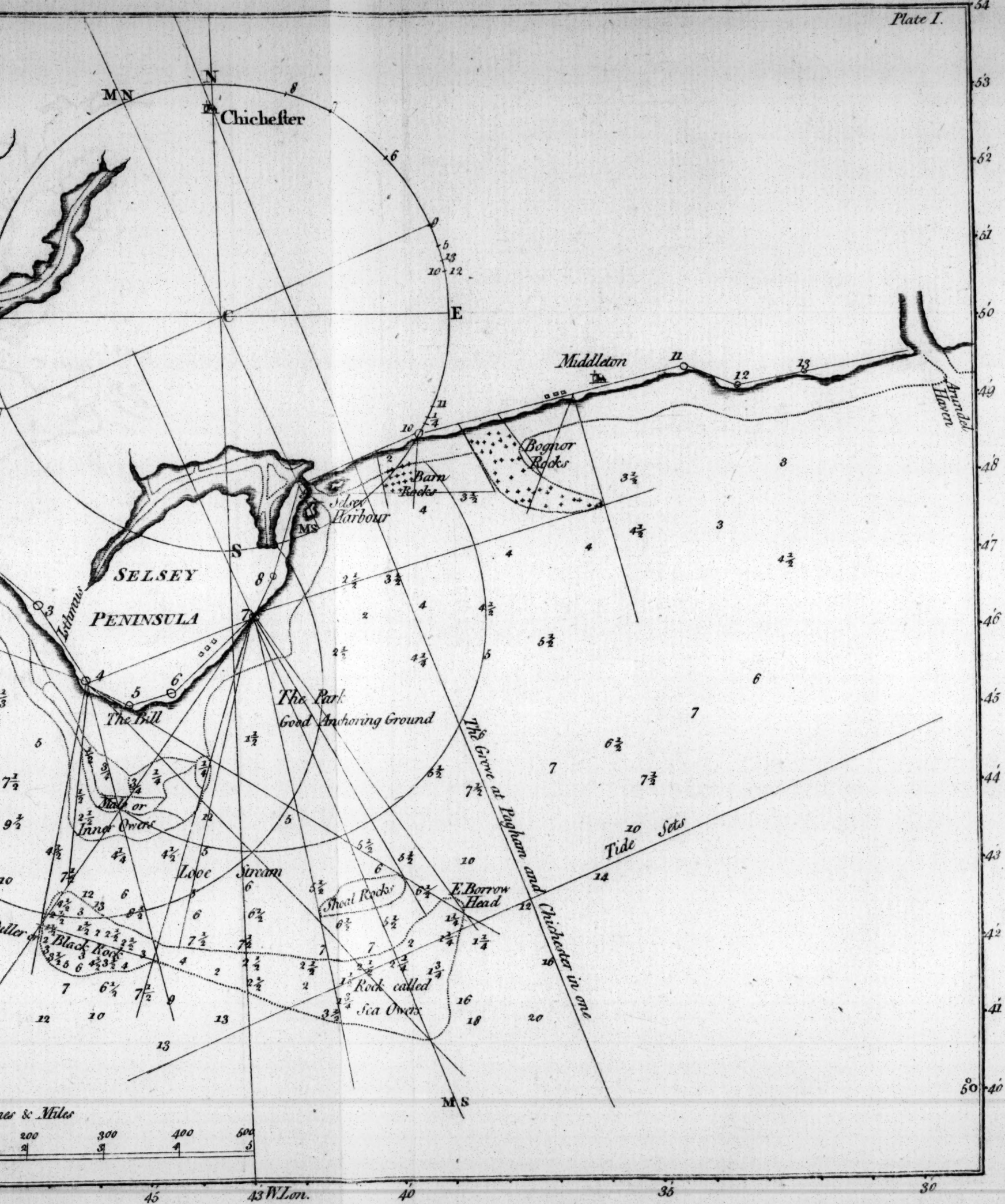
4. The angles measured with this instrument, when the objects are oblique to each other, will be too great; for in this case, the oblique angular distance is measured, and not the horizontal.

5. The point of the darts shew the course of the flowing of the tide.

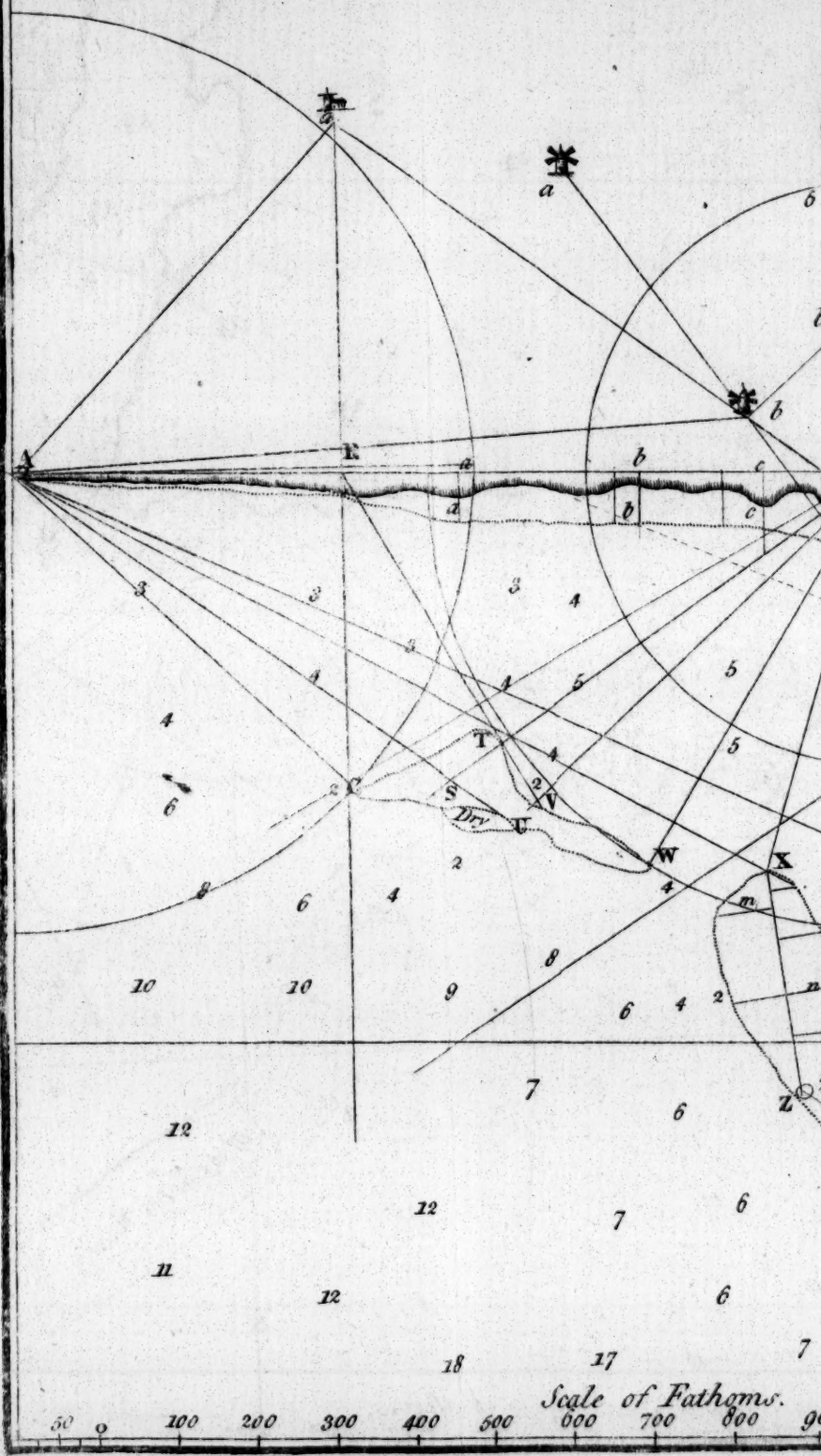
6. At the entrance of the harbour is XI, which shews the time of high water there, on the full and change days of the moon, to be at eleven o'clock.

F I N I S.





Variation 21° W.



6.072 Feet - 1 Fathom - 6^F ^{Inches} 0.864
 10 Fathoms - 1 Line = 60.864
 1000 Fathoms - 100 Lines = 1 Mile

